NONLINEAR ELLIPTIC PROBLEMS. II

BY FELIX E. BROWDER

Communicated October 25, 1963

In a preceding note [1], we proved an existence theorem for solutions of variational boundary value problems for strongly elliptic nonlinear systems of the form

$$(1) \qquad Au = \sum_{|\alpha| \leqq m} D^\alpha A_\alpha(x, u, \cdots, D^m u),$$

with A_α having at most polynomial growth, by applying a general theorem concerning nonlinear functional equations in reflexive Banach spaces. Our result in [1] extended and generalized results announced earlier by M. I. Višik [6; 7; 8] and obtained by more concrete analytic arguments. Višik's detailed account of his results which has just appeared in [9] has one feature which goes beyond the framework of methods developed in [1], namely that the hypotheses of strong ellipticity or monotonicity which are assumed involve only the variation of A_α with respect to the highest order derivatives and not the lower order derivatives of u.

It is our object in the present note to announce some results which constitute an extension of our preceding methods to cover this point. The detailed proof of these results will appear in [4].

1. We use the notation of our preceding note [1].

THEOREM 1. *Let Ω be a bounded smoothly bounded open set in R^n with boundary Γ, A a system of r differential operators of order $2m$ acting on r-vector functions $u = (u_1, \cdots, u_r)$ and having the form*

$$(1) \qquad Au = \sum_{|\alpha| \leqq m} D^\alpha A_\alpha(x, u, \cdots, D^m u).$$

Let $\zeta = \{\zeta_\alpha;\ |\alpha| \leqq m\}$, $\eta = \{\eta_\beta;\ |\beta| \leqq m-1\}$ be complex vectors. Let $E_\alpha(x, \eta, \zeta)$ be functions such that

$$A_\alpha(x, \zeta) = E_\alpha(x, \zeta, \zeta)$$

for all ζ, where E_α is measurable in x and continuous in (η, ζ). Suppose that

$$|E_\alpha(x, \eta, \zeta)| \leqq c\left\{ \sum_{|\gamma| \leqq m} |\zeta_\gamma|^{p-1} + \sum_{|\gamma| \leqq m-1} |\eta_\gamma|^{p-1} + 1 \right\}$$

for a given exponent $p > 1$.