ON THE NUMBER OF CLOSED GEODESICS ON A RIEMANNIAN MANIFOLD

BY WILHELM KLINGENBERG

Communicated by S. S. Chern, November 19, 1963

To Kurt Reidemeister on his 70th birthday

1. Let M be a compact riemannian manifold. It is a classical problem to find a lower bound for the number of closed geodesics on M, i.e., of curves which can be parametrized as a geodesic segment which has the same point as initial and as end point and has no corner there.

The only result known to hold for arbitrary manifolds M is that there is at least one closed geodesic on M, cf. Fet [**5**], Švarc [**14**], Olivier [**11**]. To get a more precise result it will presumably always be necessary to make assumptions on the topological structure of M. So it is known, e.g., that a nontrivial element of the fundamental group of M does determine a closed geodesic which represents this element. The case which has been studied most thoroughly so far is that M is diffeomorphic to the usual m-sphere S^m. After preliminary results of Poincaré [**12**], Birkhoff [**2**], Lusternik-Schnirelmann [**7**], Morse [**9**; **10**] and Lyusternik [**8**], a rather complete answer was obtained by Al'ber [**1**]. He assumes (as also Morse does in his work) that the diffeomorphism $\phi: M \to S^m$ satisfies the following condition: There is a $c > 0$ such that

$$(*) \qquad c\|d\phi X\| \leqq \|X\| < 2c\|d\phi X\|$$

for any tangent vector X to M. Here, $\|Y\|$ denotes the length of the vector Y. Obviously, this condition may be interpreted as saying that the diffeomorphism ϕ shall not differ too much from the similarity with the factor c. Now, Al'ber's result reads as follows:

Let M be a compact riemannian manifold for which there exists a diffeomorphism $\phi: M \to S^m$ satisfying (*). Put $m = 2^k + s$ with $0 \leqq s < 2^k$. Then there exist at least $2m - s - 1$ closed geodesics on M of which none is a covering of another one. All have their length in the interval $[2\pi c, 4\pi c[$.

The algebraic number of closed geodesics is at least $(m+1)m/2$.

The last statement is due to Morse [**9**; **10**]. The algebraic number of closed geodesics is obtained when counting each geodesic with a certain multiplicity. For this we refer to Al'ber [**1**].

2. In the present paper, we announce the corresponding result for a manifold which is diffeomorphic to an arbitrary compact symmetric