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DISTRIBUTION MODULO 1 AND SETS OF UNIQUENESS

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A linear set $E \subset (0, 1)$ is said to be a set of uniqueness (set U) for trigonometric expansion if no trigonometric series exists (except vanishing identically) which converges to zero in the set CE complementary to E. Following Nina Bary we shall say that E is a set of uniqueness "in the wide sense" (set U^*) if no Fourier-Stieltjes series exists (except vanishing identically) which converges to zero in CE. If E is a *closed* set U^* it means (see [1, Vol. 1, pp. 344–359, Vol. 2, p. 160]) that E does not carry any measure whose Fourier-Stieltjes coefficients tend to zero. If E is a *closed* set U (i.e. of uniqueness "strict sense") it means that E does not carry any measure or *pseudo-measure* (cf. [2]) with coefficients tending to zero.

DEFINITION. A real sequence of numbers $\{u_\kappa\}_1^\infty$ will be said to be "badly distributed" modulo 1 if there exists at least one characteristic function $X(x)$ of open interval $\Delta \subset (0, 1)$ periodic with period 1 such that

$$\limsup_{\kappa=\infty} \frac{X(u_1) + \cdots + X(u_\kappa)}{\kappa} < \int_0^1 X(x)dx = |\Delta|$$

when $|\Delta|$ stands for the length of Δ.[1]

REMARK. It is easy to see that under this hypothesis there exists a Δ with rational end-points having the same property.

THEOREM. *Let $E \subset (0, 1)$ be a linear set such that there exists an infinite sequence of positive integers $\{n_\kappa\}_1^\infty$ increasing to infinity, with the*

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[1] The reader will convince himself that all the argument which follows is applicable in the case we suppose lim inf $> \Delta$.