

BOOK REVIEWS

Wahrscheinlichkeitsrechnung mit einem Anhang über Informationstheorie. By A. Rényi. VEB Deutscher Verlag der Wissenschaften, Berlin, 1962. 11+547 pp. DM 55.

Until recently, probability theory has been called "calculus of probabilities" in the French and German languages. This book is so called and deserved the name in the best sense of the word. It is essentially a textbook of mathematical analysis as applied to the field of probability. By this it is not implied that the measure-theoretic foundations are not given adequately and rigorously. Indeed, the book begins with axiomatic Boolean algebra including a proof of M. H. Stone's isomorphism theorem, albeit in fine print. Kolmogorov's extension theorem is also given its full treatment while the Radon-Nikodym theorem, though not proved, is discussed in some detail with examples—which is probably more helpful than reproducing a standard proof. However, the unmistakable flavor of this book is the abundance of classical analytic techniques vigorously and interestingly employed to calculate the probabilities.

The author states in the Foreword that besides a careful exposition of the basic theory he has treated especially those topics that "lie near him" and about which he has more to say. Among the less usual material may be mentioned: characterizations of the normal law, Linnik-Singer's sharpening of Cramér's theorem, central limit theorems with densities, with a random number of summands, and without replacement in sampling, mixing sequences, conditional and ratio limit theorems, the author's neat treatment of order statistics, Smirnov's theorem and its variants, and an entire appendix (64 pages) devoted largely to the definitions and analytic properties of various concepts of "information content." The numerous exercises are also rich in interesting topics such as: Euler's ϕ function, the Borel-Cantelli lemma and the strong law of large numbers for pairwise independent random variables, Laplace's method of steepest descent, Post-Widder inversion formula, the Maxwell gas law, infinite series and continued fractions expansions of real numbers.

Some sacrifices of usual or fashionable topics have to be made for these unusual ones. For example, infinitely divisible laws are only mentioned and stable laws are treated only so far as Lévy's earlier derivation of the quasi-stable case. There is no mention of martingales and Markov chains are discussed only briefly in the finite case. Stochastic processes with a continuous time parameter, or special