THE DOMAIN OF DEPENDENCE INEQUALITY FOR SYMMETRIC HYPERBOLIC SYSTEMS[1]

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1. **Introduction.** Energy propagates through space with finite speed. This is a characteristic feature of wave propagation phenomena and, indeed, it is a postulate in relativistic physics. Recently [**3**; **4**] the author demonstrated this property by means of *a priori* "domain of dependence" inequalities for solutions of initial-boundary value problems for Maxwell's equations and second order hyperbolic equations and used the inequalities to prove existence, uniqueness and regularity theorems for these problems. In this announcement the results are extended to the most general systems of linear partial differential equations that admit an energy integral, the symmetric hyperbolic systems of K. O. Friedrichs [**1**].

The systems have the form

$$(1.1)\ E_{\alpha\beta}(x)\frac{\partial u_\beta}{\partial t} = A^i_{\alpha\beta}(x)\frac{\partial u_\beta}{\partial x_i} + B_{\alpha\beta}(x)u_\beta + f_\alpha(x,t)\ (\alpha = 1, 2, \cdots, m),$$

where $E_{\alpha\beta}$, $A^i_{\alpha\beta}$ and $B_{\alpha\beta}$ are real-valued functions of $x=(x_1, x_2, \cdots, x_n) \in R^n$ and $u_\beta = u_\beta(x, t)$ and f_α are real-valued functions of x and $t \in R^1$. The summation convention is used; i.e., repeated indices i and β are summed over their ranges $(1 \leqq i \leqq n,\ 1 \leqq \beta \leqq m)$. System (1.1) may also be written in matrix notation

$$E\frac{\partial u}{\partial t} = A^i\frac{\partial u}{\partial x_i} + Bu + f,$$

where $E=(E_{\alpha\beta})$, $A^i=(A^i_{\alpha\beta})$ and $B=(B_{\alpha\beta})$ are $m\times m$ matrices and $u=(u_\alpha)$ and $f=(f_\alpha)$ are $m\times 1$ (column) matrices.

A system (1.1) is symmetric hyperbolic if E and A^i $(i=1, 2, \cdots, n)$ are symmetric and E is positive definite. The form $\eta=\frac{1}{2}E_{\alpha\beta}u_\alpha u_\beta$ is interpreted as an "energy density" (energy per unit volume). The forms $\Sigma_i = -\frac{1}{2}A^i_{\alpha\beta}u_\alpha u_\beta$ are interpreted as the components of a "Poynting vector" describing the flow of power (energy per unit area per unit time). They are related by the "conservation of energy law"

$$\frac{\partial \eta}{\partial t} + \frac{\partial \Sigma_i}{\partial x_i} = \frac{1}{2}\left(B_{\alpha\beta} + B_{\beta\alpha} - \frac{\partial A^i_{\alpha\beta}}{\partial x_i}\right)u_\alpha u_\beta + f_\alpha u_\alpha.$$

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