NOTE ON LINEAR DIFFERENCE EQUATIONS

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Particular solutions of nonlinear differential equations have been used successfully to achieve analytic simplification of systems of linear differential equations [7; 8]. In this note we will show that similar results are possible for systems of linear difference equations. To the author's knowledge, this is the first time this technique has been employed for difference equations.

We are concerned with the system of linear difference equations

$$y(x+1) = x^{\mu}A(x)y(x), \tag{1}$$

where y is a vector with n components, μ is an integer, and $A(x)$ is an n by n matrix with elements analytic in a neighborhood of $x = \infty$:

$$A(x) = \sum_{s=0}^{\infty} A_s x^{-s}, \qquad |x| > \rho, \; A_0 \neq 0.$$

The most effective manner for determining the solutions formally[3] is to reduce the difference equation (1) into k systems of the same type and of lower order by a formal transformation[4] of the form

$$y(x) = T(x)z(x) \tag{2}$$

where

$$T(x) = \sum_{s=0}^{\infty} T_s x^{-s} \text{ (formally), det. } T_0 \neq 0.$$

More precisely speaking, let the resulting equation be

$$z(x+1) = C(x)z(x)$$

where $T(x)$ has been constructed so that $C(x)$ has the block diagonal form

$$C(x) = (C_1(x), C_2(x), \cdots, C_k(x)),$$

with

$$C_i(x) = x^{\mu_i} \sum_{j=0}^{\infty} C_{ij}x^{-j}, \qquad C_{i0} = \lambda_i I_i + N_i.$$

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[3] For the direct construction of formal solutions see [1; 2; 3].

[4] For the construction of the formal transformation and the resulting canonical form see [9; 10].