A COMPLEX OF PROBLEMS PROPOSED BY POST

BY WILSON E. SINGLETARY[1]

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There are eight theorems contained in this note. Theorems 1 and 7 are solutions of problems concerning fragments of the propositional calculus which were proposed by Tarski in 1946 at the Princeton Bicentennial. These problems were solved by Post and Linial[2] in 1948. Theorems 2, 4–6 and 8 are solutions of a complex of problems proposed by Post to Boone in the spring of 1948. Theorem 3 is the solution of a problem which was suggested to the writer as a natural consequence of the proof of Theorem 2. The writer became acquainted with these problems of Post while working under the direction of Boone. The methods used in the detailed proofs of these theorems, which will be published at a later date, were inspired by M. K. Yntema's[3] detailed and elegant proofs of the Post-Linial Theorems. Yntema's work was in turn suggested by an outline of the proofs given by Davis.[4] The writer understands from Boone that other detailed proofs of the Post-Linial Theorems will be forthcoming from Ronald Harrop and M. D. Gladstone. The proofs of these theorems are outlined here for completeness, because their machinery is an integral part of the proofs of the other theorems, and also because they differ somewhat from those of Yntema. All proofs of the present theorems are constructive.

A *partial propositional calculus* is a system having $\sim$, $\supset$, [, and] as primitive symbols along with propositional variables p_1, q_1, r_1, p_2, q_2, r_2, $\cdots$. Its well-formed formulas are (1) a propositional variable, (2) $[A \supset B]$, where A and B are well-formed formulas, and (3) $\sim A$ where A is a well-formed formula. It has a finite set of tautologies as axioms and its two rules of inference are modus ponens and substitution. If the restriction requiring all of the axioms to be tautologies be dropped, the resulting system will be called a *generalized partial propositional calculus.*

A *semi-Thue system*[5] is specified by a finite alphabet $\mathfrak{Z}$ and a finite set of word pairs $\mathfrak{U}$.

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[2] See [3].

[3] See [4].

[4] See [2, pp. 137–142].

[5] There are two viewpoints possible concerning semi-Thue systems. The one given here was chosen because the rules more closely resemble the axioms of the generalized partial propositional calculi used. For the two formulations see [1] and [4]. These are easily shown to be equivalent as pointed out in [4].