EMBEDDINGS IN THE TRIVIAL RANGE[1]

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1. Introduction. The fundamental problem of embedding theory is the

EMBEDDING PROBLEM. *Given a polyhedron P^k and an n-manifold M^n, classify the nice*[3] *embeddings of P^k into M^n under equivalence by (ambient) isotopy.*

This problem projects in a natural way into the

HOMOTOPY PROBLEM. *Given a polyhedron P^k and an n-manifold M^n, classify the continuous maps of P^k into M^n under equivalence by homotopy.*

It is the object of this note to report[4] that in the so-called "trivial" range of dimensions, $2k+2\leqq n$, there is a *local, topological* definition of "nice embedding" for which the above projection is bijective, and hence for which the two problems are equivalent.

Corresponding equivalences are well known in piecewise linear and differential topology [**1**; **2**], and are proved there by means of general position arguments. In the unrestricted topological case, where one considers all possible embeddings of P^k into M^n, the desired results are hopelessly false. For example [**3**], there are many nonequivalent embeddings of the closed interval $[0, 1]$ into Euclidean n-space R^n, $n\geqq 3$.

The main theorems of the present paper are

THEOREM 1.1. *Let f be a locally tame embedding of the polyhedron P^k into the combinatorial manifold M^n. If $2k+2\leqq n$, then for each $\epsilon>0$ there is an ϵ-push h of $(M^n, f(P^k))$ such that*

$$hf\colon P^k \to M^n$$

is piecewise linear with respect to arbitrary preassigned triangulations of P^k and M^n.

THEOREM 1.2. *If $2k+2\leqq n$, then for each $\epsilon>0$ there is a $\delta>0$ such that if f and f' are any two locally tame embeddings of the polyhedron P^k*

[1] A result similar to Theorem 8.1 of this paper has been obtained independently by Charles Greathouse (this Bulletin, pp. 820–823).

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[3] Definitions will be found in §2.

[4] Missing details will appear elsewhere.