# FOURIER TRANSFORMS OF SURFACE-CARRIED MEASURES AND DIFFERENTIABILITY OF SURFACE AVERAGES

BY WALTER LITTMAN[1]

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1. **Introduction.** We are given an $n$ surface $S$ (possibly with boundary) embedded in $R^{n+1}$ and a smooth mass density $\mu$ on the surface, vanishing near the boundary. We consider the following transformation of $C(R^{n+1}) \to C(R^{n+1})$:

$$g(y) = \int_{x \in S} f(y - x)\mu(x)dS_x. \tag{1}$$

Letting $D^m$ be a generic symbol for differentiation of order $m$, we ask the

QUESTION: When does there exist an estimate of the type

$$\|D^m g\|_{L_p} \leqq \text{constant} \cdot \|f\|_{L_p}? \tag{2}$$

This is related to the behavior at $\infty$ of the Fourier transform of the measure $\mu$. Our main result in that direction is the following:

ESTIMATE OF FOURIER TRANSFORMS. Let $S$ be a sufficiently smooth compact $n$-surface (possibly with boundary) embedded in $R^{n+1}$, $\mu$ a sufficiently smooth mass distribution on $S$ vanishing near the boundary of $S$. Suppose that at each point of $S$, $k$ of the $n$ principal curvatures are different from zero. Then

$$I \equiv \int_{X \in S} e(X \cdot Y)\mu(X)dS_X = O(|Y|^{-k/2}). \tag{3}$$

(Notation: $e(\cdot) \equiv e^{i(\cdot)}$.)

For the case of $\mu \equiv 1$ and surfaces of strictly positive Gaussian curvature this result has been proved by C. S. Herz [2] and previously by E. Hlawka [3]. Herz assumes $S$ to be of differentiability class $C^{[(n/2)+2]}$. For simplicity we shall not keep track in this note of the smoothness assumptions on $S$ and $\mu$. The proof here, as it stands, does not give the best results in that direction. However, it can be modified (at the expense of making it somewhat more complicated) so as to get results reducing to those of [2] in the case of positive curvature.

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