FLOWS ON SOLVMANIFOLDS

BY L. AUSLANDER AND L. GREEN[1]

Let G be a connected, simply connected solvable Lie group and let C be a closed subgroup such that G/C is compact. Further, let $g(t)$ be a one parameter subgroup of G. Then $g(t)$ induces, acting to the left on G/C, a one parameter group of transformations. We will call G/C a representation of a compact solvmanifold and the flow generated by $g(t)$ a G-induced flow. In [1] and [2] results concerning special G-induced flows were discussed in detail or research announced. It is the purpose of this note to state one aspect of the problem of G-induced flows and outline the solution that is now available. Full details and discussions will be presented elsewhere.

Let G/C be a representation of a compact solvmanifold. Our main problem may be stated as follows: Give a necessary and sufficient condition for the existence of an *ergodic* G-induced flow on G/C. We will now outline our solution to this problem.

Algebraic preliminaries. Let $L(G)$ denote a solvable Lie algebra which may be taken over the real or complex fields. Then $L(G)$ is said to be of Type (R) if all the roots of the algebra are either 0 or pure imaginary. One of the main algebraic facts we will need is the following:

THEOREM. *Given a solvable Lie algebra $L(G)$ there exists a unique minimal ideal H such that $L(G)/H$ is Type (R).*

We will also need the companion concept of a really regular element of $L(G)$. $X \in L(G)$ will be called really regular if it is
(1) regular,
(2) ad (X) has a maximal number of eigenvalues with nontrivial real part. The set of really regular elements is dense in $L(G)$.

THEOREM. *G/C has a G-induced ergodic flow if and only if there is a really regular element X of $L(G)$ such that $\exp(tX)$ induces an ergodic flow.*

REDUCTION THEOREM. *A really regular element in $L(G)$ induces an ergodic flow on G/C if and only if it induces an ergodic flow on $\mathrm{Cl}(G/CH)$, where H is the unique minimal analytic normal subgroup of G such that G/H is Type (R) and Cl denotes the closure operation.*