MAXIMAL WEDGES OF SUBHARMONIC FUNCTIONS

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The application of the Perron-Wiener method of solving the Dirichlet problem for a diffusion operator, using only subharmonic functions in the domain of the operator, may fail since the domain of the operator may be too small. This leads to the consideration of extensions of the operator to enlarge the class of subharmonic functions. If maximal classes of subharmonic functions are used in the Perron-Wiener method, then it is possible to show that the Dirichlet problem for certain regular sets is solvable for a continuous boundary function if and only if the upper and lower Dirichlet solutions are continuous functions. With one exception, proofs will be omitted since details will appear in a forthcoming paper [1].

Let $C(X)$ be the usual Banach space of real bounded continuous functions on a separable locally compact metric space X. P will denote a class of "subharmonic" functions in $C(\overline{W})$, where W is an open subset of X with compact closure $\overline{W}$ and nonempty boundary W', with the following properties: (i) P contains the constant functions; (ii) $P-P$ is dense in $C(\overline{W})$; (iii) P is a wedge in $C(\overline{W})$; and (iv) if U is a nonempty open subset of W with $\overline{U}\subset W$ and nonempty boundary U' and $f\in P$, then $f(\eta)\leqq \sup_{U'} f$ for all $\eta\in\overline{U}$. A nonempty open set U with $\overline{U}\subset W$ and nonempty boundary U' is called a regular set if $U'=\nabla_P U$ where $\nabla_P U$ is the Choquet boundary of U relative to P (see [3] for the definition and existence of the Choquet boundary). It is assumed that the regular sets form a basis for the relative topology of W.

A wedge $R\subset C(\overline{W})$ is said to be compatible with P if $R\supset P$ and $f\in R$, U regular, $\eta\in\overline{U}$ implies $f(\eta)\leqq \sup_{U'} f$. The class of wedges compatible with P can be partially ordered by set inclusion and contains a maximal element Q by Zorn's lemma. Just as in [2], it can be shown that for each $\eta\in\overline{U}$, where U is regular, there is a unit measure μ with support $S(\mu)\subset U'$ such that $f(\eta)\leqq \int f d\mu$ for all $f\in Q$. $M(\eta, U, Q)$ will denote the set of all such μ. The maximal wedge Q possesses properties not possessed by P in general. Not only is it true that Q is closed under the operation of taking the maximum of two elements of Q, but it is also true that Q is closed under an operation of taking the partial maximum of two elements of Q; i.e., if f, $g\in Q$ and there are open sets U, V with $\overline{U}\subset V\subset W$, $f\geqq g$ on U, $f\leqq g$ on $V-U$, then the function which is equal to f on U and equal to g on $\overline{W}-U$ is an