Aggregate von vorgegebenen Potenzen, Arkiv för Matematik, Astronomi och Fysik **17** (1922–1923), no. 9, 1–30.

3. S. N. Mergelyan, *Uniform approximation to functions of a complex variable*, Amer. Math. Soc. Transl. **101** (1954), 2–99.

COLUMBIA UNIVERSITY,
UNIVERSITY OF ILLINOIS, AND
UNIVERSITY OF MICHIGAN

DOUBLY INVARIANT SUBSPACES OF ANNULUS OPERATORS

BY DONALD SARASON[1]

Communicated by P. R. Halmos, April 29, 1963

1. Introduction. Let C be the unit circle in the complex plane and let C_0 be the circle $\{z: |z| = r_0\}$, where r_0 is a positive real number less than unity. The set $C \cup C_0$ is the boundary of the annulus $A = \{z: r_0 < |z| < 1\}$. Let us endow the circles C and C_0 with Lebesgue measure of total mass unity, and denote by $L^2(\partial A)$ the L^2 space associated with the measure thereby defined on the set $C \cup C_0$. This note concerns the invariant subspaces of the position operator on the space $L^2(\partial A)$, that is, of the operator Z on $L^2(\partial A)$ defined by $(Zx)(z) = zx(z)$.

We may regard $L^2(\partial A)$ as the direct sum of the two spaces $L^2(C)$ and $L^2(C_0)$. As subspaces of $L^2(\partial A)$, the latter reduce the operator Z. The restriction of Z to $L^2(C)$ is a well-known operator, a so-called bilateral shift (of unit multiplicity). The invariant subspaces of this operator have been extensively studied by Beurling [**1**], by Helson and Lowdenslager [**3**], and by Halmos [**2**]. The restriction of Z to $L^2(C_0)$ is a bilateral shift multiplied by the scalar r_0, and so has the same invariant subspace structure as a bilateral shift. The operator Z is therefore the direct sum of two operators whose invariant subspaces have been completely described. However, the problem of determining the invariant subspaces of Z involves more than merely a routine extension of known results about bilateral shifts, and as yet has not been solved completely.

[1] Research supported in part by the National Science Foundation. The results announced in this paper constitute a portion of the author's University of Michigan Doctoral Dissertation. I am deeply indebted to Professor Paul Halmos for the help he has given me over the past year.