MAXIMAL FUCHSIAN GROUPS

BY LEON GREENBERG[1]

Communicated by Lipman Bers, February 25, 1963

1. DEFINITIONS. Let D be the unit disk $\{z \mid |z| < 1\}$ and let $\mathfrak{L}$ be the group of conformal homeomorphisms of D. A Fuchsian group is a discrete subgroup of $\mathfrak{L}$. We shall be concerned here with the finitely generated Fuchsian groups. It is known that these have the following presentations.

Generators: $a_1, b_1, \cdots, a_g, b_g, e_1, \cdots, e_k, h_1, \cdots, h_m, p_1, \cdots, p_r$.

Defining relations: $e_1^{\nu_1} = e_2^{\nu_2} = \cdots = e_k^{\nu_k} = 1$,

$$\left(\prod_{i=1}^{g} a_i b_i a_i^{-1} b_i^{-1}\right) e_1 \cdots e_k h_1 \cdots h_m p_1 \cdots p_n = 1.$$

A group of the above type will be denoted $F(g; \nu_1, \cdots, \nu_k; m; n)$. The elements h_i and p_j are not distinguishable in the abstract group F, but depend on the imbedding of F in $\mathfrak{L}$. The elements h_i are hyperbolic, p_j are parabolic, and these correspond respectively to the boundary curves and punctures in the Riemann surface D/F. These elements h_i, p_j and their conjugates in F will be called *the boundary elements of F*.

We shall say that a finitely generated Fuchsian group F is *finitely maximal* (*f-maximal*) if there does not exist any other Fuchsian group G such that $F \subset G$ and the index $[G: F]$ is finite. We note that if F does not have any hyperbolic boundary elements, then F is f-maximal if and only if there does not exist any other Fuchsian group which contains it. On the other hand, if F does have hyperbolic boundary elements, then there always exist Fuchsian groups G which contain F with infinite index.

By a *geometric isomorphism* (*g-isomorphism*) of a Fuchsian group F, we shall mean an isomorphism $\gamma: F \rightarrow \mathfrak{L}$, such that

(1) $\gamma(F)$ is a Fuchsian group.

(2) γ maps the hyperbolic (parabolic) boundary elements of F onto the hyperbolic (parabolic) boundary elements of $\gamma(F)$. Let $\Gamma(F)$ denote the set of g-isomorphisms of F. $\Gamma(F)$ can be topologized in the following way. Let $f_1, \cdots, f_n$ be a set of generators for F. $\Gamma(F)$ can be imbedded in $\mathfrak{L}^n$ by assigning to $\gamma \in \Gamma(F)$ the point $(\gamma(f_1), \cdots, \gamma(f_n)) \in \mathfrak{L}^n$. $\Gamma(F)$ is given the relative topology in $\mathfrak{L}^n$. We introduce an equivalence relation ρ in $\Gamma(F)$. Let $\mathfrak{L}'$ denote the

[1] This work has been supported by the Office of Naval Research.