DIRECT DECOMPOSITIONS OF ALGEBRAIC SYSTEMS[1]

BY PETER CRAWLEY AND BJARNI JÓNSSON

Communicated by Leon Henkin, February 19, 1963

Preliminaries. An operator group with a principal series can obviously be written as a direct product of finitely many directly indecomposable admissible subgroups, and the classical Wedderburn-Remak-Krull-Schmidt theorem asserts that this representation is unique up to isomorphism. Numerous generalizations of this theorem are known in the literature.[2] Thus it follows from the results in Baer [1; 2] that if the admissible center of an operator group G satisfies the minimal and local maximal conditions, then any two direct decompositions of G (with arbitrarily many factors) have isomorphic refinements. In a somewhat different direction, it is shown in Crawley [3] that if an operator group G has a direct decomposition each factor of which has a principal series, then any two direct decompositions of G have isomorphic refinements. The results announced here yield sufficient conditions for a group (with or without operators) to have the isomorphic refinement property, one consequence being a common generalization of the two theorems just mentioned. A more detailed treatment, giving proofs, will appear shortly in [4].

Actually our results hold for a much wider class of algebraic systems, and it is in this more general framework that the theory is developed. The terminology to follow is largely the same as that in Jónsson-Tarski [6], and therefore it will be described only briefly. By an *algebra* we shall mean a system consisting of a set A, a binary operation $+$ called addition, a distinguished element 0 called the zero element of the algebra, and operations $F_t(t\in T)$ each of which is of some finite rank $\rho(t)$, subject only to the following conditions: (i) A is closed under the operation $+$ and the operations F_t $(t\in T)$; (ii) for all $x\in A$, $x+0=0+x=x$; (iii) for all $t\in T$, $F_t(0, \cdots, 0)=0$. The set T and the function ρ are assumed to be the same for all algebras under consideration. We shall identify an algebra with the set of all its elements, and shall use the same symbols, $+$, F_t, and 0, to denote the operations and the zero elements of all the algebras. If no auxiliary operations F_t are present, i.e. if $T=\phi$, then we refer to A as a *binary algebra*. An obvious example of an algebra is an operator group, i.e. an algebra for which addition is associative, each element has an

[1] This work was partially supported by National Science Foundation grants NSF G-17957 and NSF G-19673.

[2] A fairly complete list of references is given in Baer [1; 2] and Specht [8, p. 449].