COHOMOLOGY OF HOMOGENEOUS SPACES[1,2]

BY PAUL F. BAUM

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Various authors have studied the following problem: "Let K be a field or the integers. If G is a compact connected Lie group and U is a closed connected subgroup how can the cohomology of the homogeneous space G/U, $H^*(G/U; K)$, be computed from $H^*(G; K)$, $H^*(U; K)$ and some algebraic topological invariant of the way U is imbedded in G?"

The most comprehensive results to date on this question have been obtained by H. Cartan [3] and A. Borel [1]. H. Cartan [3] solved the problem for the special case when the coefficient ring is the real numbers. A. Borel [1] essentially solved the problem for the special case when U is a subgroup of maximal rank and both $H^*(G; K)$ and $H^*(U; K)$ are exterior algebras on generators of odd degree. Indeed, Borel's work in [1], together with a result of R. Bott [2], gives a complete solution for this case.

For the invariant of the imbedding of U in G both Cartan and Borel take the cohomology map $\rho^*: H^*(B_G; K)\to H^*(B_U; K)$ induced by the map $\rho: B_U\to B_G$ of classifying spaces arising from the inclusion $U\subset G$. If $H^*(G; K)$ and $H^*(U; K)$ are both exterior algebras on generators of odd degree the results of [1] give a method for computing ρ^* from group-theoretic information on how U is imbedded in G.

Using unpublished results of S. Eilenberg and J. C. Moore the following generalization of the Cartan-Borel results is obtained:

THEOREM. *Let K be a field or the integers. Assume that $H^*(G; K)$ and $H^*(U; K)$ are exterior algebras on generators of odd degree. Consider $H^*(B_U; K)$ to be an $H^*(B_G; K)$ module by means of the map $\rho^*: H^*(B_G; K)\to H^*(B_U; K)$. Then the algebra structures in $H^*(B_G; K)$ and $H^*(B_U; K)$ induce an algebra structure in*

$$\mathrm{Tor}_{H^*(B_G;K)}(K, H^*(B_U; K))$$

such that for a suitable filtration of the algebra $H^*(G/U; K)$

$$\mathrm{Tor}_{H^*(B_G;K)}(K, H^*(B_U; K)) = E_0H^*(G/U; K).$$

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[2] The work announced here is contained in the author's doctoral thesis, submitted to Princeton University. The author thanks his advisers J. C. Moore and N. E. Steenrod for the guidance and encouragement they gave him.