ON THE PLANCHEREL THEOREM OF THE 2×2 REAL UNIMODULAR GROUP

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1. **Introduction.** Let G be the group of the 2×2 real unimodular matrices, and C_c^∞ the family of all indefinitely differentiable functions on G with a compact support. It is known that there exists a family T of equivalence classes of irreducible unitary representations of G, the members of which, using the notations of [1], we denote by $C_q^{(j)}$, D_l^+ and D_l^- resp. ($j=0, \frac{1}{2}, \frac{1}{4}<q<+\infty, l=\frac{1}{2}, 1, \frac{3}{2}, \cdots$) with the following properties. (1) For any representation $U(a)$ in T and any $f\in C_c^\infty$ the operator $U_f=\int_G f(a)U(a)d\mu(a)$ is of trace class; here $d\mu(a)$ is the element of a fixed left invariant Haar measure on G (for the normalization to be used cf. below) and the trace, considered as a linear operation on C_c^∞, is a distribution. (2) Putting $T_\lambda^{(+)}(f)$ ($T_\lambda^{(-)}(f)$) for this distribution if $U(a)$ belongs to the class $C_q^{(0)}$ ($C_q^{(1/2)}$ resp., $q=\frac{1}{4}+\lambda^2$, $\lambda>0$), and $T_l(f)$ if $U(a)$ is the direct sum of a representation of class D_l^+ with a representation of class D_l^-, we have

$$(1)\qquad \begin{aligned} f(e) = {} & \int_0^\infty \tanh \pi\lambda T_\lambda^{(+)}(f)d\lambda + \int_0^\infty \lambda \coth \pi\lambda T_\lambda^{(-)}(f)d\lambda \\ & + \sum_{j=1}^\infty \frac{j-1}{2} T_{j/2}(f). \end{aligned}$$

Here e is the unit element of G. (1) is called the Plancherel formula for G.

Proofs for (1) were outlined by V. A. Bargmann [1, cf. esp. §13, p. 638] and Harish-Chandra [4]. The purpose of the present note is to suggest a new approach as a special case of a more general method to be applied later to the investigation, begun in [2], of the discrete series ($D_l^\pm$ for G) of Lorentz groups of higher dimension. Before giving the details we sketch for later use a slightly modified version of Bargmann's proof. All necessary properties of the representations of T can easily be verified through the realizations described below in §3.

We consider the subgroup G_1 of rotations of G, and put

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[1] This work was done during the author's stay at Stanford University.