ON THE BOUNDARY VALUES OF HOLOMORPHIC FUNCTIONS IN WEDGE DOMAINS

BY ADAM KORÁNYI[1]

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Let R^n be real n-dimensional Euclidean space. We call a domain $D \subset R^n$ a *regular cone* if it is nonempty, open, convex, and such that if $y \in \bar{D}$ and $y \neq 0$ then $\lambda y \in \bar{D}$ for all $\lambda > 0$ but $-y \notin \bar{D}$. Let $C^n = R^n \oplus iR^n$ be the complexification of R^n. In C^n we define the *wedge domain* corresponding to D by

$$W_D = \{z = x + iy \mid x \in R^n, y \in D\}.$$

The distinguished boundary of W_D is the set $\{z = x + iy \mid x \in R^n, y = 0\}$, i.e., just R^n.

In this paper we consider the following problem. Let f be a real function on R^n satisfying certain regularity conditions. What are the conditions for the existence of a holomorphic function F on W_D such that f is the limit of Re F as the variable approaches the distinguished boundary?

An answer to this question was given by Hans Lewy in [2] for the case where D is the positive quadrant in R^2, i.e., where W_D is equal to the product of two half planes. We are going to generalize Lewy's result in two directions: We shall consider the case of an arbitrary wedge domain, and we shall considerably relax the regularity conditions put on f and F in [2]. This additional generality is made possible by our method of proof, which consists in a systematic use of Fourier transform theory.

We shall prove two theorems. Theorem 1 is the simplest formulation that can be proved by our method. Theorem 2 is of a more general nature (although it does not seem to imply Theorem 1). The basic idea of its proof is, however, the same, despite the technical complications involved by the Fourier transform theory of distributions.

We denote by L^1 and L^2 the spaces of integrable, resp. square-integrable functions on R^n. As usual, we denote by $H^2(W_D)$ the subspace of L^2 consisting of the functions of form $\lim_{y \to 0} F_y$ where $F(z) = F(x+iy) = F_y(x)$ is holomorphic in W_D and $\{F_y \mid y \in D\}$ is a bounded set in L^2. Let D^* be the dual cone of D, i.e., the subset of the dual space of R^n consisting of the elements α such that $\langle \alpha, x \rangle > 0$ for all $x \in \bar{D}$. It is known (Bochner [1]) that $f \in H^2(W_D)$ if and only if f is

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