EXAMPLES OF DIRECT PRODUCTS OF SEMIGROUPS OR GROUPOIDS[1]

BY TAKAYUKI TAMURA

Communicated by George E. Forsythe, December 31, 1962

1. **Introduction.** The direct product $G_1\times \cdots \times G_n$ of groupoids G_i's is defined by $G_1\times \cdots \times G_n = \{(x_1, \cdots, x_n); x_i \in G_i, i=1, \cdots, n\}$ where $(x_1, \cdots, x_n)=(y_1, \cdots, y_n)$ means $x_i=y_i$ $(i=1, \cdots, n)$ and $(x_i, \cdots, x_n)(y_1, \cdots, y_n)=(x_1y_1, \cdots, x_ny_n)$.

If a semigroup A contains a subsemigroup which is isomorphic onto B, we say that A contains B.

Let i_t be one of $1, \cdots, n$, and let $i_t\neq i_s$ if $t\neq s$. $G_{i_1}\times \cdots \times G_{i_m}$, $1\leqq m<n$, is called a partial product with length m of $G_1\times \cdots \times G_n$.

It is familiar that if G_i's are groups, their direct product contains every partial product; but this is not true in the case of groupoids, not even in the case of semigroups. We can show the examples of direct product which contain no partial product. Such a direct product is called a completely exclusive direct product.

THEOREM 1. *$G_1\times \cdots \times G_n$ is a completely exclusive direct product of groupoids $G_1, \cdots, G_n$ if and only if no partial product with length $n-1$, $G_1\times \cdots \times G_{i-1}\times G_{i+1}\times \cdots \times G_n$ is homomorphic into G_i $(i=1, \cdots, n)$.*

COROLLARY 1. *If $G_1\times \cdots \times G_n$ is completely exclusive, then a partial product with length >1 is also completely exclusive.*

2. **Example for groupoids.** Let G_n be a set of n elements and ϕ be a cycle of the n elements, i.e., a cyclic permutation. The product of elements of G_n is defined by:

$$ab = (a)\phi \qquad \text{for all } a, b \in G_n.$$

Such a groupoid G_n is called a cyclic left constant groupoid. A cyclic left constant groupoid is uniquely determined by n within isomorphism, and we see that a cyclic left constant groupoid has neither idempotent element nor proper subgroupoid, and that if $m_1, \cdots, m_k$ are relatively prime in pairs and if $G_{m_1}, \cdots, G_{m_k}$ are cyclic left constant groupoids of order $m_1, \cdots, m_k$ respectively, then $G_{m_1}\times \cdots \times G_{m_k}$ is also a cyclic left constant groupoid.

[1] This paper was delivered at the meeting of the American Mathematical Society, at Santa Barbara, California, November 18, 1961. See Notices Amer. Math. Soc. **8** (1961), 513. The precise proof will be given elsewhere.