2. J. Nielson, *Untersuchungen zur Topologie der geschlossenen Zweiseitigen Flächen*, Acta Math. **50** (1927), Satz 11, 266.

3. R. Baer, *Isotopie von Kurven auf orientierbaren geschlossenen Flächen und ihr Zusammenhang mit der topologischen Deformation der Flächen*, Journ. f. Math. **159** (1928), 101.

4. W. Mangler, *Die Klassen von topologischen Abbildungen einer geschlossenen Fläche auf Sich*, Math. Z. **44** (1938), 541, Satz 1, 2,542.

5. R. Fox, *On the complementary domains of a certain pair of inequivalent knots*, Nederl. Akad. Wetensch. Proc. Ser. A **55** = Indag. Math. **14** (1952), 37–40.

6. L. Neuwirth, *The algebraic determination of the topological type of the complement of a knot*, Proc. Amer. Math. Soc. **12** (1961), 906.

INSTITUTE FOR DEFENSE ANALYSES

THE PRODUCT OF A NORMAL SPACE AND A METRIC SPACE NEED NOT BE NORMAL

BY E. MICHAEL[1]

Communicated by Deane Montgomery, January 16, 1963

An old—and still unsolved—problem in general topology is whether the cartesian product of a normal space and a closed interval must be normal. In fact, until now it was not known whether, more generally, the product of a normal space X and a metric space Y is always normal. The purpose of this note is to answer the latter question negatively, even if Y is separable metric and X is Lindelöf and hereditarily paracompact.

Perhaps the simplest counter-example is obtained as follows: Take Y to be the irrationals, and let X be the unit interval, retopologized to make the irrationals discrete. In other words, the open subsets of X are of the form $U \cup S$, where U is an ordinary open set in the interval, and S is a subset of the irrationals.[2] It is known, and easily verified, that any space X obtained from a metric space in this fashion is normal (in fact, hereditarily paracompact). Now let Q denote the rational points of X, and U the irrational ones. Then in $X \times Y$ the two disjoint closed sets $A = Q \times Y$ and $B = \{(x, x) \mid x \in U\}$ cannot be separated by open sets. To see this, suppose that V is a neighborhood of B in $X \times Y$. For each n, let

$$U_n = \{x \in U \mid (\{x\} \times S_{1/n}(x)) \subset V\},$$

---

[1] Supported by an N.S.F. contract.

[2] The usefulness of this space X for constructing counterexamples was first called to my attention, in a different context, by H. H. Corson.