A LINEAR INITIAL-VALUE PROBLEM

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The problem before us today is connected with the role of the Stieltjes integral concept as related to linear differential systems. For example, suppose that P is a numerical function defined on the real line and Lebesgue integrable on each interval, and each of c and Y is a real number. It is commonplace that the function F, absolutely continuous on each interval, described by the differential requirement

$$(1) \qquad F(c) = Y \quad \text{and} \quad F' = F \cdot P \text{ almost everywhere,}$$

is equivalently described by integrating both sides of the differential equation from the initial point c (the integration being in the sense of Lebesgue); another description, however, is provided by the ordinary Stieltjes integral requirement

$$(2) \qquad F(z) = Y + \int_c^z F \cdot d\phi \text{ for each real number } z,$$

where ϕ is any function, absolutely continuous on each interval, having P as its almost everywhere derivative. Similar translation is feasible, of course, with finite systems of first order equations: Y and the functions F and P and ϕ are taken to be matrix valued, and juxtaposition is then interpreted as matrix multiplication.

In connection with such systems of first order linear equations, there are "interface problems" wherein prescribed discontinuities are imposed on the otherwise locally absolutely continuous function F in equation (1) (the 1955 work of F. W. Stallard [14] and of T. J. Pignani and W. M. Whyburn [11] is basic in that area). Some types of interface singularities can also be introduced in these systems *via* the function ϕ in equation (2); recently Stallard [15] has succeeded in translating some of these modified versions of (2) back into the differential equation setting. I regret that his important work does not fall within the scope of my subsequent remarks here today.

I wish to focus attention on possible modifications of the integral in system (2) which may arise when the continuity condition on ϕ is dropped (a bounded variation condition being retained), and on related modifications in the "adjoint system," which, in the continuous case, inherits the form,

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