MODELS OF COMPLETE THEORIES

R. L. VAUGHT

The semantical concepts, such as satisfaction, truth, and model, form the subject matter of a field known as the theory of models. I am going to discuss today several related recent developments in this field. They all lie in one particular area which is indicated by the title and which will be described more fully in a moment. However, some introductory and side remarks I shall make may also serve to indicate to those unfamiliar with the theory of models at least what some of the other areas of the field are.

Perhaps the earliest result in the theory of models, dating from 1915, is the theorem of Löwenheim and Skolem: *Any infinite algebraic system has a denumerable subsystem having the same true* (*elementary*) *sentences*. Before discussing this theorem further, we must define the notions involved in it.

By an *algebraic system* is meant a system $\mathfrak{A} = \langle |\mathfrak{A}|, R_0^{\mathfrak{A}}, R_1^{\mathfrak{A}}, \cdots \rangle$ formed by a nonempty set $|\mathfrak{A}|$ and finitely or denumerably many relations $R_0^{\mathfrak{A}}, R_1^{\mathfrak{A}}, \cdots$ among the elements of $\mathfrak{A}$, each $R_n^{\mathfrak{A}}$ having a finite number ρ_n of places. Thus, for example, an ordered group is a system $\langle G, <, \cdot, e\rangle$ having a binary relation, a binary operation (which may be regarded as a special kind of ternary relation), and a distinguished element (a special kind of singulary relation). A system $\mathfrak{B}$, having the same similarity type ρ as $\mathfrak{A}$, is a *subsystem* of $\mathfrak{A}$ if $|\mathfrak{B}| \subseteq |\mathfrak{A}|$ and each $R_n^{\mathfrak{B}}$ is $R_n^{\mathfrak{A}}$ restricted to $|\mathfrak{B}|$. The cardinal number of $\mathfrak{A}$ $(\overline{\mathfrak{A}})$ means that of $|\mathfrak{A}|$.

The symbols of the elementary language L_ρ are the sentential connectives $\wedge, \vee, \sim, \rightarrow, \leftrightarrow$, the quantifiers $\forall$, $\exists$, the individual variables $v_0, v_1, \cdots$, the equality symbol $\approx$, and the ρ_n-ary relation symbols $R_0, R_1, \cdots$. A typical *formula* ϕ (of L_ρ), taking $\rho_0 = 2$, is

$$\forall v_1(P_0 v_0 v_1 \vee v_0 \approx v_1),$$

which has v_0 as its only free variable. An example of a *sentence* σ (of L_ρ), i.e., of a formula with no free variables, is

$$\exists v_0 \forall v_1(P_0 v_0 v_1 \vee v_0 \approx v_1).$$

It is clear what we mean by saying that σ is *true* in $\mathfrak{A}$, or $\mathfrak{A}$ is a *model* of σ; namely, in this case, that $\mathfrak{A}$ has a kind of first element. Similarly,

An address delivered before the Vancouver meeting of the Society on August 30, 1962, by invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings; received by the editors December 21, 1962.