

ON FINITE GROUPS AND THEIR CHARACTERS

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The idea of a presidential address seems to require a lecture delivered in the most refined and dignified scientific atmosphere yet understandable to the layman, a lecture which treats a difficult field of mathematics in such a complete manner that the audience has the excitement, the aesthetic enjoyment of seeing a mystery resolved, perhaps only with the slightly bitter feeling, of asking afterwards: Why did I not think of that myself?

Well, I don't know how my predecessors did it, but I know that I can't do it. Since the founding fathers of the Society have placed the presidential address at the time in the life of the president when he disappears into anonymity among the ranks of the Society, I shall not even try it. The choice of the field about which I am going to speak was a natural one for me, not only because of my own work in the theory of groups of finite order, but because of the new life which has appeared in this field in recent years. However, in spite of all our efforts, we know very little about finite groups. The mystery has not been resolved, we cannot even say for sure whether order or chaos reigns. If any excitement can be derived from what I have to say, it should come from the feeling of being at a frontier across which we can see many landmarks, but which as a whole is unexplored, of planning ways to find out about the unknown, even if the pieces we can put together are few and far apart. My hope then is that some of you may go out with the idea: "Now let me think of something better myself."

Let me first mention one difficulty of the theory. We have not learned yet how to describe properties of groups very well; we lack an appropriate language. One of the things we can do is to speak about the characters of a group G . I cannot define characters here. Let me only mention that we have a partitioning of the group G into disjoint sets K_1, K_2, \dots, K_k , the classes of conjugate elements. The characters then are k complex-valued functions χ_1, \dots, χ_k , each constant on each class K_i . They have a number of properties which connect them with properties of the group. These characters can be used to prove general theorems on groups, but we seem to have little control about what can be done and what not. You will see this more clearly later when I discuss specific results. I can give two reasons

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