STABLE STRUCTURES ON MANIFOLDS

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Communicated by Deane Montgomery, October 2, 1962

Recent developments in topology seem to support the observation that a number of essentially local problems in certain manifolds can be solved by ignoring the local nature of the problem and instead using to advantage the global form of the manifold.

It is the object of this report to describe a certain structure on manifolds, called a *stable structure*, which capitalizes on this observation. Orientable differentiable and piecewise linear manifolds, as well as all simply connected topological manifolds, support stable structures, and one can solve in such manifolds a number of problems, both local and global in nature, which remain unsolved in general.

The three parts of this report correspond to the papers [**1**; **2**; **3**], in which the various details may be found. The first part deals with the homeomorphisms of the n-sphere and describes some results necessary for the development of the machinery of stable structures. The second part introduces stable structures and shows that certain theorems proved for the n-sphere will hold for an arbitrary connected topological manifold if and only if that manifold supports a stable structure. The third part applies the machinery of stable structures to some problems in the field of topological manifolds.

I. HOMEOMORPHISMS OF S^n

1. Definitions. The set of points $\{(x_1, \cdots, x_n): \sum x_i^2 \leqq 1\}$ in Euclidean n-space R^n will be denoted by D^n and its boundary by S^{n-1}. D^n and any space homeomorphic to D^n will be called a *closed n-cell.* S^{n-1} and any space homeomorphic to S^{n-1} will be called an *$n-1$-sphere.*

$H(S^n)$ will denote the group of homeomorphisms of S^n onto itself.

A k-manifold M^k in an n-manifold M^n will be said to be *locally flat* if each point of M^k has a neighborhood U in M^n such that the pair $(U, U\cap M^k)$ is topologically equivalent to the pair (R^n, R^k). Then $\text{Hom}(S^{n-1}, S^n)$ will denote the set of all locally flat embeddings of S^{n-1} into S^n.

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[1] The first named author gratefully acknowledges the support of the National Science Foundation and the Institute for Advanced Study. The second named author gratefully acknowledges the support of the National Academy of Sciences and the Air Force Office of Scientific Research.