RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

ANALYTIC MEASURES ON COMPACT GROUPS

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Communicated by Ralph Phillips, August 14, 1962

The purpose of this note is the announcement of an extension to compact abelian groups of the two celebrated theorems of F. and M. Riesz [8] concerning analytic measures on the circle group. The content of these theorems is as follows:

Let μ be a Borel measure on the circle satisfying

$$\int_{-\pi}^{+\pi} e^{in\theta} d\mu(\theta) = 0, \qquad n = 1, 2, 3, \cdots .$$

Then

A. *μ is absolutely continuous with respect to Lebesgue measure and*

B. *If μ vanishes identically*[2] *on a set of positive Lebesgue measure, then μ must be the zero measure.*

It is not hard to see that A and B together are equivalent to the following:

The collection of Borel sets on which μ vanishes identically is invariant under rotation.

This is the assertion concerning analytic measures that we extend to compact groups. We also shall state several of its consequences, including analogues of A and B. The work was inspired by, and is in part an extension of, several of the results of Helson and Lowdenslager [4; 5].

In all that follows G is a compact abelian group,[3] $\hat{G}$ its discrete dual, and ψ is a fixed homomorphism of $\hat{G}$ into the group R of real numbers. The mapping $\psi: \hat{G} \rightarrow R$ is a continuous homomorphism and

[1] Supported in part by National Science Foundation Grant G14779 and the United States Air Force Office of Scientific Research.

[2] We say that a measure μ *vanishes identically* on a set E if μ vanishes on all Borel subsets of E.

[3] See however our final remarks.