ON EXTENDING CROSS SECTIONS IN ORIENTABLE $V_{k+m,m}$ BUNDLES

BY MARK MAHOWALD[1]

Communicated by W. S. Massey, August 2, 1962

1. **Introduction.** 1.1. Let $\mathfrak{b}=(F, p, B)$ be an orientable n-plane bundle, i.e., $p(\mathfrak{b})$: $F(\mathfrak{b})\rightarrow B(\mathfrak{b})$ is a bundle map with R^n as the fiber, $SO(n)$ as the group and a connected C.W. complex $B(\mathfrak{b})$ as base space. Let $w_i(\mathfrak{b})\in H^i(B(\mathfrak{b}); J)$ be the Stiefel-Whitney classes of $\mathfrak{b}$. The group $J=Z$ if i is odd or $i=n$ and J equals Z_2 otherwise; $w_n(\mathfrak{b})$ is also the Euler class of $\mathfrak{b}$.

1.2. Associated with $\mathfrak{b}$ are $V_{n,m}$ bundles, $\mathfrak{b}^m$ ($V_{n,m}$ is the Stiefel manifold of orthogonal m frames in R^n). Letting $k=n-m$ we will write

$$\mathfrak{b}^m = (F_{k,m}(\mathfrak{b}), B(\mathfrak{b}), p(\mathfrak{b})).$$

We are interested in finding invariants which would tell whether or not there exists a cross section in $\mathfrak{b}^m$. In a sense this problem is already solved using the Postnikov systems. What we will do is to identify certain classes in universal examples whose image in $H^*(B(\mathfrak{b}))$ must be zero if a cross section is to exist over the $k+6$ skeleton of $B(\mathfrak{b})$. The computations are based on a modification and extension of the results of Hermann [2].

The first obstruction is just $w_{k+1}(\mathfrak{b})$. It turns out that the higher classes are not unique elements but rather cosets of certain groups. We can specify this group for each obstruction studied in terms of computable operations in $H^*(B)$. In addition, these higher classes satisfy certain relations. By using both the indeterminacy and these relations, the obstructions can be computed in many interesting cases. Our detailed computation for the first six obstructions are valid only in the stable range for the homotopy groups of $V_{k+m,m}$. Full details will appear elsewhere.

Some of the applications of these results to the question of immersing and embedding manifolds into Euclidean space are listed in §5.

2. **Obstruction theory.** 2.1. Let $\mathfrak{g}=(E, p, G_{k+m})$ be the universal $k+m$ plane bundle over G_{k+m}, the Grassman manifold of oriented $k+m$ planes in R_∞. Let $\mathfrak{g}^m=(E_{k,m}, p, G_{k+m})$ be the associated $V_{k+m,m}$ bundle. Let $\mathfrak{b}=(F, p, B)$ be any $k+m$ plane bundle. To each bundle

[1] This work was supported by a grant from the U. S. Army Research Office (Durham).