THE EXPONENTIAL DECAY OF SOLUTIONS OF THE WAVE EQUATION IN THE EXTERIOR OF A STAR-SHAPED OBSTACLE

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In this paper we study the behavior for large time of solutions of the wave equation in three space dimensions in the exterior of some smooth, bounded reflecting obstacle which is assumed to be star-shaped. We shall prove that, given an initial disturbance, the bulk of its energy is propagated to infinity. The precise statement of the result is Theorem IV; it has as its corollary the following result:

Let u denote a smooth solution of the above exterior problem whose initial values have finite energy and are zero outside of some bounded region. Then at a fixed point x, u(x, t) decays exponentially with time.

When the scattering obstacle is a sphere, this result has been deduced by Wilcox [5] by analyzing the explicit expression for the solution obtained by separation of variables. In [4], C. S. Morawetz has proved that energy decays like the inverse square of time, and it follows that the solution decays like $1/t$. The sharper result of the present paper is obtained by combining her result with the techniques developed in [1] and [2].

Theorem IV of this paper implies that the scattering matrix associated with the above problem can be continued analytically from the lower half-plane into a horizontal strip of the upper half-plane; a complete discussion of this can be found in [2].

H_0 denotes the Hilbert space of Cauchy data $\phi = (\phi_1, \phi_2)$ defined in the entire three-dimensional space, normed by the energy norm:

$$\|\phi\|^2 = \int ((D\phi_1)^2 + \phi_2^2)dx.$$

$U_0(t)$ denotes the operator which relates the data at time t to the data at time zero of solutions of the wave equation in all of space. H denotes the subspace of data which vanish inside the obstacle and $U(t)$ denotes the operator which relates the data at time t to data at time zero of solutions of the wave equation in the exterior of the obstacle which vanish on the obstacle for all time. $U_0(t)$ and $U(t)$ are unitary operators mapping H_0 and H^0, respectively, onto themselves (conservation of energy) and they form one-parameter groups.

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