ON THE PRODUCT OF A CONTRACTIBLE TOPOLOGICAL MANIFOLD AND A CELL

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Communicated by Edwin Spanier, June 22, 1962

1. **Introduction.** Our object is to prove the following theorem:

THEOREM. *Let M be a contractible manifold of dimension m.*
(A) *If M has empty boundary, then $M \times R^p \approx R^{m+p}$ for some p.*
(B) *If M is compact, then $M \times D^q \approx D^{m+q}$ for some q.*

For a bound on p and q, see §4.

Here D^i denotes the closed unit disk in Euclidean i-space R^i, and $X \approx Y$ means X is homeomorphic to Y. Our manifolds are not assumed to have any differential or combinatorial structures unless this is explicitly indicated. Throughout the paper M denotes a manifold of dimension m, ∂M is the boundary of M, and int $M = M - \partial M$.

2. **Proof of (A).** The analog of (A) for combinatorial manifolds is due to McMillan [4]. In view of this, to prove (A) it suffices to establish that $M \times R^j$ is homeomorphic to an open subset of R^{m+j} for some j. The following result of Curtis and Lashof [3] is used for this purpose.

(C) *Let $U \subset M \times M$ be a neighborhood of the diagonal. Let ϕ: $U \rightarrow R^m$ be a map such that for all $y \in M$, $\phi(y, y) = 0$ and $\phi | U_y$ is one-one, where $U_y = U \cap (M \times y)$. Then $M \times R^j$ can be embedded in R^{m+j} for some j.*

In the terminology of Milnor's theory of microbundles, the existence of ϕ is equivalent to the triviality of the *tangent microbundle* of M. In his paper [5], Milnor deals with combinatorial microbundles, but as he points out, much of the development applies to the topological case. In particular, it is true that homotopic maps induce equivalent microbundles. Therefore the tangent microbundle of a contractible manifold is trivial. Thus (C) may be used to complete the proof of (A).

3. **Proof of (B).** If M is contractible so is int M. Thus (A) applies to int M, and to prove (B) it suffices to demonstrate the following.

(D) *Let M be contractible. If* (int M) $\times R^k \approx R^{m+k}$, *then* $M \times D^{k+1} \approx D^{m+k+1}$.

The proof depends on some work of M. Brown.

(E) *For any M, there is a neighborhood of ∂M in M homeomorphic to* $(\partial M) \times D^1$.

This is proved in [2].

Put $S^{n-1} = \partial D^n =$ unit sphere in R^n.