PRODUCTS OF PSEUDO CELLS

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By a *pseudo n-cell* is meant a contractible compact combinatorial n-manifold with boundary (whose boundary is not necessarily an $(n-1)$-sphere). Poenaru [6] and Mazur [5] gave the first examples of pseudo 4-cells which are not topological 4-cells, and Curtis [4] has shown that, for each $n \geqq 4$, there exists a pseudo n-cell which is not a topological n-cell because its boundary fails to be simply connected. By a *homotopy n-cell* is meant a pseudo n-cell whose boundary is the $(n-1)$-sphere S^{n-1}. It follows from the generalized Poincaré conjecture and the generalized Schoenflies theorem that a homotopy n-cell is a topological n-cell if $n \geqq 5$ [4].

The following consequence of theorems of Brown and Stallings generalizes results of Curtis [4], who has shown that the cartesian product of a pseudo n-cell and an interval is the topological $(n+1)$-cell I^{n+1} if $n \geqq 5$, and Andrews [1], who has shown that the product of a homotopy 3-cell with I^3 and the product of a homotopy 4-cell with I^2 are both I^6.

THEOREM. *If M^p and N^q are pseudo cells of positive dimensions p and q respectively, with $p+q \geqq 6$, then*[2] $M^p \times N^q = I^{p+q}$.

COROLLARY. *If $n \geqq 8$, then I^n is the product of two combinatorial manifolds with boundary, neither of which is a topological cell.*

The following lemma is perhaps well known, but it does not seem to have appeared in print.

LEMMA. *If C^n is a compact n-manifold with boundary such that* Int $C^n = E^n$ *(euclidean n-space) and* $B = \text{Bd } C^n = S^{n-1}$, *then* $C^n = I^n$.

PROOF. By Brown's result that the boundary of a manifold is collared [3], there is a homeomorphism h of $B \times [0, 1]$ into C^n such that $h(x, 0) = x$ if $x \in B$. Then, by the generalized Schoenflies theorem [2], the collared $(n-1)$-sphere $h(B \times 1/2)$ bounds a closed n-cell A in Int $C^n = E^n$. Hence $C^n = A \cup h(B \times [0, 1/2])$ is a closed n-cell.

PROPOSITION. *If C^n is a compact combinatorial n-manifold with boundary, $n \geqq 6$, with* Int $C^n = E^n$, *then* $C^n = I^n$.

PROOF. By the Lemma it suffices to show that the boundary B of C^n is an $(n-1)$-sphere. By the generalized Poincaré conjecture, it is therefore sufficient to show that $\pi_i(B)$ is trivial for $0 \leqq i < n-1$ [7; 9].

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[2] Equality here denotes topological equivalence.