

results for operators not assumed positive by means of a reduction procedure [4] and the present theorems.

We are indebted to the work of Eberhard Hopf for suggesting that a resolution of this type is possible.

#### BIBLIOGRAPHY

1. R. V. Chacon and D. S. Ornstein, *A general ergodic theorem*, Illinois J. Math. **4** (1960), 153–160.
2. R. V. Chacon, *The influence of the dissipative part of a general Markov process*, Proc. Amer. Math. Soc. **11** (1960), 957–961.
3. ———, *Identification of the limit of operator averages*, J. Math. Mech. (to appear).
4. ———, *Convergence of operator averages*, (to appear).

BROWN UNIVERSITY

## TCHEBYCHEFF QUADRATURE IS POSSIBLE ON THE INFINITE INTERVAL<sup>1</sup>

BY J. L. ULLMAN

Communicated by J. L. Doob, May 29, 1962

The purpose of this announcement is to state a theorem on Tchebycheff quadrature which answers a question posed in [1], and to discuss the proof. Complete details will appear elsewhere.

### 1. Tchebycheff quadrature.

DEFINITION 1.1. A unit mass distribution on  $(-\infty, \infty)$  possessing moments of all positive integer order will be said to belong to class  $D$ .

DEFINITION 1.2. Let  $\psi$  be an element of  $D$  and  $n$  a positive integer. We refer to the equations

$$\frac{1}{n} \sum_{i=1}^n x_{i,n}^k = \int x^k d\psi, \quad k = 1, \dots, n$$

as the equations  $(\psi, n)$ . These equations admit a solution  $x_{1,n}, \dots, x_{n,n}$  which is unique up to permutation of the first index.

DEFINITION 1.3.  $T$  quadrature is said to be possible for an element  $\psi$  of  $D$  if equations  $(\psi, n)$  have real solutions for every positive integer  $n$ . If  $T$  quadrature is possible for  $\psi$  it is called a  $T$  distribution.

<sup>1</sup> This research was supported in part by National Science Foundation Grant No. G19654.