# RESOLUTION OF POSITIVE OPERATORS

BY R. V. CHACON[1]

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We begin with a $\sigma$-finite measure space $(S, \mathfrak{F}, \mu)$ and a positive linear operator $T$ on $L_1$ with $\|T\|_1 \leqq 1$. The main result of this note is the following theorem:

THEOREM 1. *If $p \in L_1$ and is non-negative and if $T$ is regular, then*

$$L_1 \overset{p}{=} I_p \oplus \overline{M}^2$$

*where*

$$M = \{g: g = f - Tf, f \in L_1\},$$
$$I_p = \{hp: T(hp) = hTp, hp \in L_1\}.$$

Before proceeding we give some definitions, in particular that of a *regular operator*. We have used this term in the statement of Theorem 1.

DEFINITION. *$T$ is regular if*

$$\sum_{k=0}^{\infty} T^k p(s) = +\infty$$

*whenever $p \in L_1$ is strictly positive.*

We summarize results [2], reducing general operators to regular operators in the following theorem:

THEOREM 2. *There exists a set $S_1$ so that the restriction of $T$ to $L_1(S_1, \mathfrak{F}_1, \mu)$, where $\mathfrak{F}_1 = \mathfrak{F} \cap S_1$, is regular and so that $\sum_{k=0}^{\infty} |T^k f| < +\infty$ on $c\ S_1$. Furthermore, for each $f \in L_1(S_1, \mathfrak{F}_1, \mu)$ there exists a $f_{S_1} \in L_1(S_1, \mathfrak{F}_1, \mu)$ so that*

$$\lim_{n \to \infty} \frac{\sum_{k=0}^{n} T^k (f - f_{S_1})}{\sum_{k=0}^{n} T^k p} = 0$$

*on $S_1 \cap \{s: \sum_{k=0}^{\infty} T^k p(s) > 0\}$.*

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[2] $A =^p B$ if the spaces are equal with respect to the seminorm obtained by integration over the support of $\sum_{k=0}^{\infty} T^k p(s)$.