RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

ADDITIVITY OF THE GENUS OF A GRAPH

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In this note a graph G is a finite 1-complex, and an imbedding of G in an orientable 2-manifold M is a geometric realization of G in M. The letter G will also be used to designate the set in M which is the realization of G. Manifolds will always be orientable 2-manifolds, and $\gamma(M)$ will stand for the genus of M. Given a graph G the *genus* $\gamma(G)$ of G is the smallest number $\gamma(M)$, for M in the collection of manifolds in which G can be imbedded.

A *block* of G is a subgraph B of G maximal with respect to the property that removing any single vertex of B does not disconnect B. (A block with more than two vertices is a "true cyclic element" in Whyburn [3].) Given G there is a unique finite collection $\mathfrak{B}$ of blocks B of G such that $G = \bigcup B$, $B \in \mathfrak{B}$. The collection $\mathfrak{B}$ is called the *block decomposition* of G. If G is connected and $\mathfrak{B}$ contains k blocks; then they may be listed in an order $B_1, \cdots, B_k$ such that

$$(1) \qquad \bigcup_1^j B_i \text{ is connected, and } B_{j+1} \cap \bigcup_1^j B_i \text{ is a vertex of } G \text{ for } j = 1, \cdots, (k-1).$$

A *2-cell imbedding* of G is an imbedding in a manifold M such that each component of $(M-G)$ is an open 2-cell. (See Youngs [4]). The *regional number* $\delta(G)$ of a graph G is the maximum number of components of $(M-G)$ for all possible 2-cell imbeddings of G. In [4] it was shown that if G is connected then

$$(2) \qquad \delta(G) = 2 - \chi(G) - 2\gamma(G)$$

where $\chi(G)$ is the Euler characteristic of G.

The object of this note is to prove two formulas about the block decomposition of a connected graph G with k blocks $B_1, \cdots, B_k$:

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