ON ANALYTICITY AND PARTIAL DIFFERENTIAL EQUATIONS

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Introduction. It is the purpose of the present note to present in outline some results established in a forthcoming paper of the writer [2] on solutions of a family of partial differential equations depending upon a parameter. The questions treated were suggested (though somewhat obliquely) by a reading of the paper by Kodaira and Spencer [4] concerning solutions of differentiable families of elliptic differential operators on a vector bundle over a compact manifold, with applications to problems concerning variation of complex structure. The results with which we are concerned in [2] relate essentially to the drastically different case of noncompact manifolds. For the sake of simplicity in the present exposition, we shall restrict ourselves to differential operators operating on scalar functions on an open subset of Euclidean n-space.

Let G be an open subset of the Euclidean n-space E^n, M_1 a real-analytic manifold. We consider a family $\{A_t\}$ of differential operators with (possibly) variable coefficients on G, with the coefficients depending also upon the parameter t in M_1. Thus in the usual notation for partial differential operators,

$$\left(D_j = i^{-1}\partial/\partial x_j,\ \alpha = (\alpha_1, \cdots, \alpha_n),\ D^\alpha = \prod_{j=1}^{n} D_j^{\alpha_j},\ |\alpha| = \sum_{j=1}^{n} \alpha_j\right),$$

$$A_t = \sum_{|\alpha| \leqq r} a_\alpha(x, t) D^\alpha.$$

We say that $\{A_t\}$ is an analytic family if each a_α is a real-analytic function on $G \times M_1$.

The questions we pose concerning such a family $\{A_t\}$ are the following:

(I) *Does there exist a family of fundamental solutions $e_t(x, y)$ for A_t on G which depend analytically upon t in M_1 and which for a given parameter value t_0 in M_1 coincide with a prescribed fundamental solution $e_0(x, y)$ for A_{t_0}?*

(II) *If f is an analytic function from M_1 into the analytic functions on G, does there exist a family of solutions $\{u_t\}$ of the equations*

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