SINGULAR PERTURBATIONS

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Communicated February 7, 1962

Let P_ϵ designate the problem of finding a solution of the differential equation

$$\epsilon y'' + F(t, y, y', \epsilon) = 0, \qquad 0 \leqq t \leqq 1, \tag{1}$$

that satisfies the boundary conditions

$$y(0) = \alpha(\epsilon), \qquad y(1) = \beta(\epsilon). \tag{2}$$

Here ϵ is a small positive parameter approaching zero. We envisage circumstances under which $y = y(t, \epsilon)$ approaches a limit nonuniformly in t as $\epsilon \to 0+$, the nonuniformity occurring at $t = 0$. Accordingly, the limiting problem P_0 involves the differential equation

$$F(t, u, u', 0) = 0, \qquad 0 \leqq t \leqq 1, \tag{3}$$

with the single boundary condition

$$u(1) = \beta(0). \tag{4}$$

Partial derivatives will be denoted by subscripts, thus $F_y = \partial F/\partial y$, etc.

For a solution $u = u(t)$ of (3) we define the function ϕ and the region D_δ by

$$\phi(t) = \int_0^t F_{y'}(\tau, u(\tau), u'(\tau), 0)\,d\tau,$$

$$D_\delta = [(t, y, y', \epsilon) : 0 \leqq t \leqq 1, \ |y - u(t)| < \delta,$$
$$|y' - u'(t)| < \delta(1 + \epsilon^{-1}e^{-\phi(t)/\epsilon}), \ 0 < \epsilon < \epsilon_0].$$

ASSUMPTIONS. (A) *The problem* P_0, (3) *and* (4), *possesses a solution* u *which is twice continuously differentiable on* $[0, 1]$.

(B) *For some* $\delta > 0$, F *possesses partial derivatives of the first and second orders with respect to* y *and* y' *in* D_δ, *and* F *as well as these partial derivatives are continuous functions of* t, y, y' (*for fixed* ϵ).

(C) $F(t, u(t), u'(t), \epsilon) = O(\epsilon)$; $q(t, \epsilon) = F_y(t, u(t), u'(t), \epsilon) = O(1)$; $p(t, \epsilon) = F_{y'}(t, u(t), u'(t), \epsilon) = \phi'(t) + \epsilon p_1(t, \epsilon)$ *where* ϕ *is twice continu-*

[1] This research was supported in part by the National Science Foundation under Grant No. NSF G-19914.