TWO APPLICATIONS OF THE METHOD OF CONSTRUCTION BY ULTRAPOWERS TO ANALYSIS

BY W. A. J. LUXEMBURG[1]

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1. **Introduction.** Recently, A. Robinson in [1] has given a proper extension of classical analysis, which he called nonstandard analysis. His theory is based on the general metamathematical result that there exist nonstandard models for the system R of real numbers. Such models of R may be constructed in the form of ultrapowers as defined by T. Frayne, D. Scott, and A. Tarski in [2]. The object of this paper is to apply Robinson's method in order to obtain a new proof of the Hahn-Banach extension theorem and in order to give a new and simple proof of a result about the existence of certain measures on Boolean algebras which was recently obtained by O. Nikodým in [3; 4].

It may be of interest to the reader to point out that the use of nonstandard arguments in the proof of the Hahn-Banach extension theorem eliminates the use of Zorn's lemma. In fact, the validity of the Hahn-Banach extension theorem is a consequence of the apparently weaker hypothesis that every proper filter is contained in an ultrafilter, i.e., the prime ideal theorem for Boolean algebras. It seems likely, that conversely the Hahn-Banach extension theorem implies the prime ideal theorem for Boolean algebras.

A more detailed presentation of the subject of this announcement will be contained in lecture notes on nonstandard analysis under preparation by the author.

2. **Nonstandard models of** R. Let R denote the real number system. Let D be an arbitrary set and let $\mathfrak{U}$ be an ultrafilter on D. If A and B are two mappings of D into R, i.e., $A, B \in D^R$, then we say that $A \equiv_{\mathfrak{U}} B$ if and only if $\{n: n \in D \text{ and } A(n) = B(n)\} \in \mathfrak{U}$. The relation $A \equiv_{\mathfrak{U}} B$ is easily seen to be an equivalence relation. The set $D^R/\mathfrak{U}$ of all equivalence classes will be denoted by R^* and the equivalence class of a mapping A of D into R will be denoted by a. Thus $A \in a$. Finally, we define the algebraic operations in R^* as follows: $a+b=c$ if and only if there exist elements $A \in a$, $B \in b$ and $C \in c$ such that $\{n: n \in D \text{ and } A(n)+B(n)=C(n)\} \in \mathfrak{U}$; and a similar definition

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