TCHEBYCHEFF APPROXIMATION IN A COMPACT METRIC SPACE

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Communicated by A. Erdélyi, March 26, 1962

1. **Introduction.** In this note[1] an outline of two theories of Tchebycheff approximation is given for functions defined on a compact metric space. The first[2] of these lacks the elegance of the one variable theory, but is descriptive of the true situation. Many results of the one variable theory have counterparts here. The second theory is developed only for functions defined on finite point sets. A special type of Tchebycheff approximation, the strict approximation, is introduced and the resulting theory is similar to the classical one variable theory. It is, in particular, shown that the strict approximations are unique. These theories allow one to solve the central problem of approximation theory, the computation of best and strict approximations.

2. **Preliminaries.** Spaces and sets in general are denoted by capital script letters $\mathcal{A}$, $\mathcal{B}$, $\cdots$ and elements of such sets are denoted by lower case letters x, y, $\cdots$. Let $\mathcal{B}$ be a compact metric space. The space of real-valued continuous functions defined on $\mathcal{B}$ is denoted by $\mathcal{C}$ and has elements f, g, $\cdots$. The norm in $\mathcal{C}$ is taken to be

$$\|f\| = \max_{x\in\mathcal{B}} |f(x)| .$$

Let $\mathcal{L}$ be an n-dimensional subspace of $\mathcal{C}$ with basis functions $g_i(x)$, $i=1, 2, \cdots, n$.

$$L(A, x) = \sum_{i=1}^{n} a_i g_i(x), \qquad |a_i| < \infty,$$

is an element of $\mathcal{L}$ with parameters $A=(a_1, a_2, \cdots, a_n)$.

The Tchebycheff approximation problem in this context is stated as follows: Given $f(x)$ in $\mathcal{C}$ determine A^* such that

$$\|f(x) - L(A^*, x)\| \leqq \|f(x) - L(A, x)\|$$

for all A. Such a $L(A^*, x)$ is said to be a *best approximation* to $f(x)$ with *deviation* $\|f(x)-L(A^*, x)\|$. The elements of $\mathcal{B}$ where the norm is assumed, i.e.,

[1] Proofs of these results and some related results will be published elsewhere. Preprints are available from the author for some of this material.

[2] A similar viewpoint has been presented recently by Lawson [5].