A NON-HOPFIAN GROUP[1]

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The purpose of this note is to construct a non-hopfian[2] and finitely generated group S which is in no way complicated. This group S is a generalised free square[3] of the free nilpotent group A of class two on two generators. Since A satisfies the maximum condition, S differs radically from the non-hopfian groups constructed by Graham Higman [**1**], with which it may be compared. It may perhaps be of interest to point out that the first finitely generated non-hopfian groups were constructed by B. H. Neumann [**2**]. Besides these and the non-hopfian groups of Higman [**1**], the only other known finitely generated non-hopfian groups are those due to B. H. Neumann and Hanna Neumann [**3**] and to P. Hall [**4**].

The construction of S is as follows. We take a replica B of A. Thus we may present A and B as follows:

$$A = gp(a, b; [a, b, b] = [a, b, a] = 1),$$
$$B = gp(c, d; [c, d, d] = [c, d, c] = 1).$$

Here, as is the custom, we define

$$[x, y] = x^{-1}y^{-1}xy, \qquad [x, y, z] = [[x, y], z], \qquad x^y = y^{-1}xy,$$

where x, y, z belong to some group G.

We now define

$$H = gp(a, [a^2, b]) \quad \text{and} \quad K = gp([c, d], c).$$

It is easy to verify that H and K are free abelian of rank two and hence isomorphic. Therefore we can form the generalised free product S of A and B amalgamating H with K:

$$S = (A * B; a = [c, d], [a^2, b] = c).$$

It is clear that we may present S as follows:

$$S = gp(a, b, d; [a, b, b] = [a, b, a] = 1,$$
$$[[a^2, b], d, d] = [[a^2, b], d, [a^2, b]] = 1, a = [a^2, b, d]).$$

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[2] A group G is hopfian if it is not isomorphic to any proper factor group of itself. If G is not hopfian, we say G is non-hopfian.

[3] A generalised free square is, by definition, the generalised free product of two isomorphic groups.