SIMPLE NEIGHBORHOODS

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The purpose of this note is to state a collection of results, all related to questions of uniqueness of smooth neighborhoods of finite complexes as imbedded (nicely) in differentiable manifolds. The reported results will appear in subsequent papers, the first of which is [1]. The approach taken is always via the theory of simple homotopy types (due to J. H. C. Whitehead), and the main theorem below (Uniqueness of simple neighborhoods) is an application of the Nonstable Neighborhood Theorem, proved in [1] (suggested by the remarkable work of Smale on high-dimensional manifolds).

What is necessary in Differential Topology is:

(I) A theorem asserting the existence of a smooth neighborhood about any finite complex, nicely imbedded in a differentiable manifold, where smooth neighborhood is taken in the strictest possible sense.

(II) A theorem asserting uniqueness of smooth neighborhoods about any fixed nicely imbedded complex, where smooth neighborhood is taken in the weakest conceivable sense.

An existence theorem in the style of (I) is proved in Chapter 7 of [1]. The term neighborhood is as given in [1]. To fulfill the ambitions of (II), the concept of simple neighborhood is introduced (Definition 2), and the Uniqueness of Simple Neighborhoods Theorem is proved.

A strengthening of the notion of h-cobordism (more suitable to the framework of this theory) is s-cobordism (Definition 3, below), and an application of the Simple Neighborhood Uniqueness theorem is the result that any s-cobordism (between manifolds of dimension greater than 4) is trivial. Hence if two such manifolds are s-cobordant, then they are differentiably isomorphic. This is a generalization to non-simply-connected manifolds of the h-cobordism theorem of Smale. See [3].

In subsequent papers this theory will be applied to the study of differentiable knots, and the setting up of an obstruction theory of the type hinted about in [2].

A differentiable isomorphism $\phi: (A, B)\to(A', B')$ for differentiable manifolds, $A\supseteq B$, $A'\supseteq B'$ will mean an isomorphism $\phi: A\to A'$ such that

$$\phi \mid B: B \overset{\to}{\approx} B'$$