DIFFERENTIABLE PERIODIC MAPS

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1. **The bordism groups.** This note presents an outline of the authors' efforts to apply Thom's cobordism theory [6] to the study of differentiable periodic maps. First, however, we shall outline our scheme for computing the oriented bordism groups of a space [1]. These preliminary remarks bear on a problem raised by Milnor [4]. A *finite manifold* is the finite disjoint union of compact connected manifolds with boundary each of which carries a C^∞-differential structure. The boundary of a finite manifold, B^n, is denoted by ∂B^n. A *closed manifold* is a finite manifold with void boundary. We now define the oriented bordism groups of a pair (X, A).

An *oriented singular manifold* in (X, A) is a map $f: (B^n, \partial B^n) \to (X, A)$ of an oriented finite manifold. Such a singular manifold *bords* in (X, A) if and only if there is a finite oriented manifold W^{n+1} and a map $F: W^{n+1} \to X$ such that $B^n \subset \partial W^{n+1}$ as a finite regular submanifold whose orientation is induced by that of W^{n+1} and such that $F | B^n = f$, $F(\partial W^{n+1} - B^n) \subset A$. From two such oriented singular manifolds (B_1^n, f_1) and (B_2^n, f_2) a disjoint union $(B_1^n \cup B_2^n, f_1 \cup f_2)$ is formed with $B_1^n \cap B_2^n = \emptyset$ and $f_1 \cup f_2 | B_i^n = f_i$, $i = 1, 2$. Obviously $-(B^n, f) = (-B^n, f)$. We say that two singular manifold (B_1^n, f_1) and (B_2^n, f_2) are *bordant* in (X, A) if and only if the disjoint union $(B_1^n \cup -B_2^n, f_1 \cup f_2)$ bords in (X, A). By the well-known angle straightening device [5] this is shown to form an equivalence relation. The oriented bordism class of (B^n, f) is written $[B^n, f]$ and the collection of all such bordism classes is $\Omega_n(X, A)$. An abelian group structure is imposed on $\Omega_n(X, A)$ by disjoint union, and then following Atiyah we refer to $\Omega_n(X, A)$ as an oriented *bordism group* of (X, A). The weak direct sum $\Omega_*(X, A) = \sum_0^\infty \Omega_n(X, A)$ is a graded right module over the oriented Thom cobordism ring Ω. For any $f: (B^n, \partial B^n) \to (X, A)$ and any closed oriented manifold V^m the module product is given by $[B^n, f][V^m] = [B^n \times V^m, g]$ where $g(x, y) = f(x)$. For any map $\phi: (X, A) \to (Y, B)$ there is an induced homomorphism $\phi_*: \Omega_n(X, A) \to \Omega_n(Y, B)$ given by $\phi_*([B^n, f]) = [B^n, \phi f]$. There is also $\partial_*: \Omega_n(X, A) \to \Omega_{n-1}(A)$ given by $\partial_*([B^n, f]) = [\partial B^n, f | \partial B^n \to A]$. Actually $\phi_*: \Omega_*(X, A) \to \Omega_*(Y, B)$ and $\partial_*: \Omega_*(X, A) \to \Omega_*(A)$ are Ω-module homomorphisms of degree 0 and -1.

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