VECTOR FIELDS ON SPHERES

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Let us write $n=(2a+1)2^b$, where a and b are integers, and let us set $b=c+4d$, where c and d are integers and $0\leqq c\leqq 3$; let us define $\rho(n)=2^c+8d$. Then it follows from the Hurwitz-Radon-Eckmann theorem in linear algebra that there exist $\rho(n)-1$ vector fields on S^{n-1} which are linearly independent at each point of S^{n-1} (cf. [**4**]).

THEOREM 1.1. *With the above notation, there do not exist $\rho(n)$ linearly independent vector fields on S^{n-1}.*

This theorem asserts that the known positive result, stated above, is best possible. Like the theorems given below, it is copied without change of numbering from a longer paper now in preparation.

Theorem 1.1 may be deduced from the following result (cf. [**1**]).

THEOREM 1.2. *The truncated projective space $RP^{m+\rho(m)}/RP^{m-1}$ is not coreducible; that is, there is no map $f\colon RP^{m+\rho(m)}/RP^{m-1}\rightarrow S^m$ such that the composite*

$$S^m = RP^m/RP^{m-1} \xrightarrow{i} RP^{m+\rho(m)}/RP^{m-1} \xrightarrow{f} S^m$$

has degree 1.

Theorem 1.2 is proved by employing the "extraordinary cohomology theory" $K(X)$ of Atiyah and Hirzebruch [**2**; **3**]. If our truncated projective space X were coreducible, then the group $K(X)$ would split as a direct sum, and this splitting would be compatible with any "cohomology operations" that one might define in the "cohomology theory" $K(X)$.

THEOREM 5.1. *It is possible to define operations*

$$\Psi^k_\Lambda\colon K_\Lambda(X)\rightarrow K_\Lambda(X)$$

for any integer k (positive, negative or zero) and for $\Lambda=R$ (real numbers) or $\Lambda=C$ (complex numbers). These operations have the following properties.

(i) *Ψ^k_Λ is natural for maps of X.*

(ii) *Ψ^k_Λ is a homomorphism of rings with unit.*

(iii) *The following diagram is commutative.*

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