ON APPROXIMATION BY SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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Introduction. In a recent note in the Bulletin [1], the writer announced some results established in [2], generalizing the theorems of Walsh and Mergelyan, on uniform approximation by solutions of elliptic differential equations of arbitrary order for which the property of uniqueness in the Cauchy problem holds. The generalization of the Walsh theorem stated in [1] was restricted to elliptic operators with constant top-order coefficients. It is the object of the present note to state some sharper results whose proofs appear in [4] among which is the complete generalization of the Walsh Theorem to elliptic operators with variable top-order coefficients. The strengthening of the technical apparatus underlying these proofs permits also the generalization of our approximation theory to some classes of non-elliptic operators, and in particular to hyperbolic operators and to the class of operators of principal normal type having strongly pseudo-convex level surfaces which have been recently studied by Hörmander [5].

In order to obtain sharper approximation theorems, we must consider domains which are more smoothly bounded than the mildly regular domains of [2], namely the following:

DEFINITION. *Let G be a precompact domain N in E^n. Then G is said to be firmly regular if for each point x_0 of* bdry(G) *there exists a neighborhood N, a constant $h>0$, and a unit vector ξ_0, $|\xi_0|=1$, such that the cone at x given by*

$$C_x = \{y : y = x + r\xi,\ 0 < r < h,\ |\xi| = 1,\ |\xi - \xi_0| < h\}$$

is completely contained in G for each x in $N \cap \bar{G}$.

The basic property of firmly regular open sets which underlies our approximation theorems is the following:

LEMMA 1 (LEMMA 8 OF [4]). *Let G be an open subset of E^n, G_1 a firmly regular open subset with compact closure in G, A a linear differential operator of order r with coefficients in $C^1(G)$.*

(a) *Let u be an element of the Sobolev space $W^{r-1,q}(G)$ with support in $\bar{G}_1$ such that Au is a finite Radon measure on $\bar{G}_1$. (We denote the space*

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