RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

COMBINATORIAL EMBEDDINGS OF MANIFOLDS[1]

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The following results on embedding manifolds resemble in their form Dehn's Lemma, the Sphere Theorem, and, especially, embedding theorems obtained for differentiable manifolds by A. Haefliger [1].

Let M, Q be finite combinatorial manifolds of dimensions m and q, respectively. Let $\dot{M}$, $\dot{Q}$ be their boundaries (possibly empty), and let $f: M \to Q$ be a piecewise linear map. We define *sing* (f) to be the closure in M of the set $\{x \in M; f^{-1}f(x) \neq x\}$. Let $R = \dot{M} \cap f^{-1}(\dot{Q})$, S be a regular neighbourhood of R in $\dot{M}$ (see [3]) and $T = \dot{M} - \mathring{S}$.

THEOREM 1. *Of the following conditions,* (i), (ii), (iii), *and any one of* (iv), (v), (vi) *are sufficient to ensure the existence of a piecewise linear embedding* $g: M \subset Q$ *such that* g *is homotopic to* f *rel.* $\dot{M}$:

(i) $q \geqq m+3$,

(ii) M *is* $(2m-q)$ *connected,*

(iii) Q *is* $(2m-q+1)$ *connected,*

(iv) $f(\dot{M}) \subset \dot{Q}$,

(v) $sing(f) \cap \dot{M} = \emptyset$ *and* T *is* $(3m-2q+1)$ *connected,*

(vi) $sing(f) \cap R = \emptyset$ *and* T *is* $(2m-q-1)$ *connected.*

REMARKS. If $\dot{M} = \emptyset$, we regard condition (iv) as being trivially satisfied. If $f(\dot{M}) \subset \mathring{Q}$, we have the convention that the only regular neighbourhood of the empty set is the empty set, and so $T = \dot{M}$.

In particular:

COROLLARY 2. *Any element of* $\Pi_m(Q)$, *where* Q *is* $(2m-q+1)$ *connected* $(q \geqq m+3)$, *may be represented by a piecewise linear embedding of* S^m.

THEOREM 3. *If* $q = 2m$, *there exists a piecewise linear embedding*

[1] This is an abstract of a thesis to be submitted for the degree of Doctor of Philosophy at Cambridge University. The research was carried out under Dr. E. C. Zeeman and maintained by a D.S.I.R. grant.