RESEARCH PROBLEMS

1. Frank Harary: *Matrix theory.*

What is the maximum value of the determinant among all square matrices of order n:

(a) with entries 0 or 1 (*binary matrices*)?

(b) with entries -1 or $+1$?

(c) with entries -1, 0, or $+1$?

At the International Symposium on Matrix Computation, April 24–28, 1961, the first question was asked by E. Aparo and the other two by L. Collatz. It was pointed out that the answer to (b) is known when $n=2^m$.

Classical Function Theory Problems

A Colloquium on Classical Function Theory was held at Cornell University August 17–21, 1961 with support from the National Science Foundation.

The following is a list of problems drawn up by members of the Colloquium. The list contains both old and new problems. The notations are those used in Nevanlinna's book "Eindeutige analytische Funktionen." (Received October 16, 1961.)

2. If $f(z)$ is entire and $\lim_{r\to\infty} \log M(r)r^{-\rho}$ exists, does $\lim T(r)r^{-\rho}$ exist?

3. Let $n_1(r, a)$ denote the number of simple zeros of $f(z)-a$ in $|z| \leqq r$. Conjecture: If $f(z)$ is an entire function of finite order ρ and $n_1(r, a)=O(r^c)$, $n_1(r, b)=O(r^c)(c<\rho) a\neq b$, $a, b\neq\infty$, then ρ is an integer multiple of $1/2$.

4. Functions satisfying $|f(z)| \leqq Me^{k|z|}$ in the plane and $|f(x)| \leqq A$ on the real axis are known to satisfy $|f(z)| \leqq Ae^{k|y|}$. What conclusion can be reached if the condition $|f(x)| \leqq A$ is replaced by $|f(x)| \leqq A$ on the positive real axis and $|f(x)| \leqq B$ on the negative axis? The real axis may be divided in various other ways and similar questions asked. What part does the constant M play? If $M<A$ we evidently have additional information.

5. Is there a bounded analytic function defined in $|z|<1$ such that $N(r, f')/(-\log(1-r))\to 1$ as $r\to 1$? Can such an example be constructed as a gap series $f(z)=\sum c_n z^{\lambda_n}$, $\sum |c_n|<\infty$?

6. If $f(z)$ is meromorphic in $|z|<1$ and $T(1, f')<\infty$, is $T(1, f)<\infty$?