MAGNITUDE OF THE FOURIER COEFFICIENTS OF AUTOMORPHIC FORMS OF NEGATIVE DIMENSION

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1. Let Γ be an H-group, i.e., Γ is a group of linear transformations of the upper half-plane $\mathcal{H}$ on itself that is discontinuous in $\mathcal{H}$, not discontinuous at any real point, possesses translations, and admits a fundamental region bounded by a finite number of sides. Let F be regular in $\mathcal{H}$ and at the parabolic vertices of Γ, and

$$F(V\tau) = \epsilon(V)(c\tau + d)^r F(\tau) \qquad V\ (\dot{c}\dot{d}) \in \Gamma,\ \tau \in \mathcal{H}, \tag{1}$$

where ϵ is a multiplier system for Γ and $-r$. Then F has a Fourier series:

$$F(\tau) = \sum_{m=0}^{\infty} a_m e((m + \alpha)\tau/\lambda), \qquad \operatorname{Im} \tau > 0, \tag{2}$$

where $e(u) = \exp(2\pi i u)$; α and λ are defined below.

The order of magnitude of the Fourier coefficients a_m has been actively investigated for many years. Recently Petersson [2] gave estimates for forms of small negative dimension $(0<r<2)$, a range inaccessible by the usual methods. He proved:

$$a_m = O(m^{r/2}), \quad 0 < r < 2, \quad r \neq 2^{-h} \quad \text{for } h = 0, 1, 2, \cdots; \tag{3}$$

$$a_m = O(m^{r/2} \log^{r/2} m), \quad r = 2^{-h} \quad \text{for } h = 0, 1, 2, \cdots. \tag{4}$$

The object of this note is a slight improvement of these estimates. *We shall show that* (4) *is superfluous and that, in fact,*

$$a_m = O(m^{r/2}) \tag{5}$$

holds for all r in the range $0<r<2$.

2. We shall use our variant of the circle method (cf. [1]). Since we are interested in the Fourier coefficients (i.e., expansion coefficients at $i\infty$), it is necessary to modify the method slightly. (Also we write $-r$ for the dimension of the form, while in [1] we wrote r.) Select a fundamental region R_0 with cusp at $p_0 = i\infty$; denote the remaining inequivalent cusps in R_0 by $p_1, \cdots, p_s$. Let $S_0 = (1\ \lambda | 0\ 1)$, $\lambda > 0$, generate the subgroup of Γ fixing ∞, and let $\epsilon(S) = e(\alpha)$, $0 \leqq \alpha < 1$. Define λ_j and α_j correspondingly for $j = 1, \cdots, s$. We have the expansions, valid in $|t_j| < 1$, $|t| < 1$: