ON THE COHOMOLOGY OF TWO-STAGE POSTNIKOV SYSTEMS

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1. **Introduction.** The purpose of this paper is to compute the cohomology of certain spaces with two nonvanishing homotopy groups. Let $P(\pi, n; \tau, m, k)(n<m)$ denote the space with homotopy groups π and τ in dimensions n and m, all other homotopy groups equal to zero, and (first) k-invariant equal to $k \in H^{m+1}(K(\pi, n), \tau)$. Let ϵ_i be the basic class in $H^i(K(\tau, i), \tau)$. We shall then compute the mod 2 cohomology of $P_{n,h}=P(Z_2, n, Z_2, 2^h n-1, \epsilon_n^{2^h})$.

Extending the methods of this paper, further computations can be carried out. This will be done in a subsequent paper.

2. **The Steenrod construction.** In this section we are working in the category of *css*-complexes. In the (non-normalized) chain complex $C_*(K)$ of a *css*-complex K we can define a filtration. Let namely σ_q denote a q-simplex in K. We can then in a unique way write σ_q in the form

$$\sigma_q = s_{i_1} s_{i_2} \cdots s_{i_{q-p}} \sigma_p, \qquad 0 \leqq i_{q-p} < \cdots < i_1 < q,$$

where σ_p is a nondegenerate p-simplex in K and s_i denotes a degeneracy operator in K. The generator $\sigma_q \in C_q(K)$ is then said to be of filtration p

$$\sigma_q \in F_p C_*(K).$$

This defines a filtration in $C_*(K)$.

Let π be a permutation group on the n letters $(0, 1, \cdots, n-1)$ and let V be an arbitrary π-free resolution of the integers. Let V be filtered by dimension. Let $V \otimes C_*$ and $C_*^{(n)}$ (the n-fold tensor product of C_*) be filtered by the usual tensor product filtration. Let π operate trivially in C_*, diagonally in $V \otimes C_*$, and by permutation of the factors in $C_*^{(n)}$. We then have the

THEOREM. *There exists a natural π-equivariant filtration and augmentation preserving transformation*

$$(1) \qquad \phi' \colon V \otimes C_* \to C_*^{(n)}.$$

If Φ' is another such transformation then ϕ' and Φ' are homotopic by a natural π-equivariant homotopy of degree $\leqq 1$ (i.e. $H(v \otimes \eta) \in F_{p+i+1}$ if $\dim v = i$ *and $\eta \in F_p$).*