ANALYTIC CONTINUATION OF THE PRINCIPAL SERIES

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The purpose of this note is to announce results obtained in the analytic continuation of the (nondegenerate) "principal series" of representations of the $n \times n$ complex unimodular group. This study has as its starting point a similar one for the 2×2 real unimodular group previously carried out by us in [4].

We let G be the $n \times n$ complex unimodular group and C its diagonal subgroup consisting of elements $c = (c_1, c_2, \cdots, c_n)$. A continuous character λ of C is given by

$$\lambda(c) = \left(\frac{c_1}{|c_1|}\right)^{m_1} \cdots \left(\frac{c_n}{|c_n|}\right)^{m_n} |c_1|^{s_1} \cdots |c_n|^{s_n}$$

where the sequences of integers $m_1, m_2, \cdots, m_n$ and complex numbers $s_1, s_2, \cdots, s_n$ are uniquely determined by setting $0 \leqq m_1 + m_2 + \cdots + m_n < n$ and $s_1 + s_2 + \cdots + s_n = 0$. Notice that λ is unitary, i.e., has values in the circle group, if $\operatorname{Re}(s_j) = 0$, $j = 1, 2, \cdots, n$. Gelfand and Neumark have shown how to construct for each unitary λ an irreducible unitary representation $a \to T(a, \lambda)$ of the group G [2]. To describe these representations (i.e., the principal series) we follow the method but not the notation of [2].

Let V be the subgroup of G of elements having ones on the main diagonal and zeros above the main diagonal. Then G acts on V in a natural way; we denote the action of $a \in G$ on $v \in V$ by $v\tilde{a}$ (the transformations $v \to v\tilde{a}$ are linear fractional transformations when $n = 2$ and generalizations thereof in higher dimensions). The operators of the representation $T(\cdot, \lambda)$ are given by

$$T(a, \lambda)f(v) = m(v, a; \lambda)f(v\tilde{a})$$

where $m(v, a; \lambda)$ is an appropriate multiplier, and the underlying Hilbert space is $L_2(V)$.

In order to state our results we introduce a tube $\mathfrak{I}$ lying in the complex hyperplane $s_1 + s_2 + \cdots + s_n = 0$. The base B of $\mathfrak{I}$ is the smallest convex set which contains the points $(\sigma, -\sigma, 0, 0, \cdots, 0)$, $-1 < \sigma < 1$ and is invariant under all permutations of coordinates. A

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