TRANSMISSION PROBLEMS IN FUNCTION THEORY

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In the year 1908, J. Plemelj [8] published an ingenious solution of the very difficult Hilbert problem for systems of analytic functions. His proof, which utilizes the theory of Fredholm integral equations, may also be found in the appendix of Muskhelishvili's monograph on singular integral equations [7]. Since Plemelj's solution of this important problem is of a fairly complicated nature, we feel that, in the light of recent advances in the theory of elliptic partial differential equations [1; 3; 9; 10] and linear analysis [4; 5], it is worthwhile to present a new proof which, while it is perhaps technically complicated, is conceptually much simpler than that given by Plemelj. Our approach, which is based on a continuity method, will be outlined in this note. We shall in fact discuss a general class of transmission problems, which includes the Hilbert problem mentioned above. Detailed proofs will be presented in another publication.

1. **Statement of the problem.** We shall pattern our formulation of the problem after that proposed in a preceding paper [6]. Accordingly, we denote by R a given closed Riemann surface of genus h, and by L, a system $L_1, L_2, \cdots, L_N$, of simple, closed, disjoint, oriented regular curves with continuously turning tangents. Suppose that $T^+(s)$ and $T^-(s)$ are given $2n \times n$ matrix functions on L, whose entries are functions of class C^1, such that

$$(1.1) \qquad \det(T^+, \overline{T}^-) \neq 0$$

for all points on L. Let $Fd\bar{z}$ be a given square integrable $n \times 1$ conjugate matrix differential on R (i.e., $Fd\bar{z}$ is to be invariant under conformal transformations). Denoting left and right boundary values on L of an $n \times 1$ matrix function W on $R-L$ by W^+ and W^- respectively, we state our problem as follows:

Find all strong solutions of the transmission problem

$$(1.2) \qquad W_{\bar{z}} = F,$$

$$(1.3) \qquad \operatorname{Re} \overline{T}^+ W^+ = \operatorname{Re} \overline{T}^- W^-.$$

The concept of "strong solution" is defined, for example, in [10], and will become clear from the discussion below.

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