Norm ideals of completely continuous operators. By Robert Schatten. Ergebnisse der Mathematik und ihrer Grenzgebiete, Neue Folge, Heft 27. Berlin-Göttingen-Heidelberg, Springer-Verlag, 1960. viii +81 pp. DM 23.60.

Let $\mathfrak{H}$ be an infinite-dimensional complex Hilbert space, and let $\mathfrak{A}$ be the algebra of all bounded operators on $\mathfrak{H}$. An ideal (two-sided) $\mathfrak{T}$ of $\mathfrak{A}$ is made into a normed linear space by introducing a norm α on $\mathfrak{T}$. In general, the norm $\alpha(A)$ of $A \in \mathfrak{T}$ does not coincide with the bound $\|A\|$ of the operator A. A norm α is a *crossnorm*, if $\alpha(A) = \|A\|$ for all operators $A \in \mathfrak{T}$ of rank 1. α is *uniform*, if $\alpha(XAY) \leqq \|X\| \cdot \|Y\| \cdot \alpha(A)$ for all $A \in \mathfrak{T}$ and X, $Y \in \mathfrak{A}$. An ideal $\mathfrak{T}$ of $\mathfrak{A}$ is called a *norm ideal*, if there is defined on $\mathfrak{T}$ a uniform crossnorm with respect to which $\mathfrak{T}$ is a Banach space. As indicated by its title, this monograph is a study of norm ideals of $\mathfrak{A}$ formed by completely continuous operators.

After a preliminary discussion of completely continuous operators, the main part of the book begins with some general theorems on ideals of $\mathfrak{A}$: Calkin's theorem that, in case of separable $\mathfrak{H}$, every proper ideal of $\mathfrak{A}$ is contained in the ideal $\mathfrak{C}$ of all completely continuous operators; Kaplansky's theorem that if a left ideal of $\mathfrak{A}$ formed by completely continuous operators is closed in the uniform topology and annihilates only the nullvector, it must coincide with all of $\mathfrak{C}$.

Next, the reader is introduced to two well-known ideals of $\mathfrak{A}$: the Schmidt class (σc) and the trace class (τc). In a natural way, uniform crossnorms σ, τ are defined on (σc), (τc) respectively by $\sigma(A) = (\sum_j \|Ax_j\|^2)^{1/2}$ and $\tau(A) = \sum_j ((A^*A)^{1/2}x_j, x_j)$, where $\{x_j\}$ is a complete orthonormal system in $\mathfrak{H}$. With these respective norms σ and τ, (σc) and (τc) are norm ideals of $\mathfrak{A}$ formed by completely continuous operators. The Banach space (τc), with τ as norm, may be interpreted as the conjugate space $\mathfrak{C}^*$ of the Banach space $\mathfrak{C}$ of all completely continuous operators, when the operator bound is taken as the norm in $\mathfrak{C}$. The Banach space $\mathfrak{A}$ of all bounded operators on $\mathfrak{H}$, again with the operator bound as norm, may be interpreted as the second conjugate space $\mathfrak{C}^{**}$ of $\mathfrak{C}$. But $\mathfrak{C}$ is not the conjugate space of any Banach space, when $\mathfrak{H}$ is infinite-dimensional.

The remaining part of the book deals mainly with the problem of determining all minimal norm ideals of $\mathfrak{A}$. A norm ideal is *minimal*, if none of its proper subspaces is a norm ideal. Let $\mathfrak{R}$ be the ideal of $\mathfrak{A}$ formed by all operators of finite rank. A crossnorm α on $\mathfrak{R}$ is