

# LIE AND JORDAN STRUCTURES IN SIMPLE, ASSOCIATIVE RINGS

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1. **Introduction.** In classifying the simple, finite-dimensional Lie algebras over the field of complex numbers, Cartan and Killing showed that, outside of five isolated examples, these fall into four infinite families.

These four infinite families are, roughly speaking, the matrices of trace 0 and the skew-symmetric matrices under some involution defined for the algebra of matrices.

It is natural to ask whether the simplicity of these sets of matrices, as Lie algebras, is really a consequence of the simplicity of the algebra of all  $n \times n$  matrices over a field, as an associative ring.

We shall see that this is indeed the case. The simplicity of an associative ring, and by this we mean any simple ring, with no chain conditions assumed (it can even be a radical ring) forces the simplicity or almost-simplicity of certain natural nonassociative structures obtained from the elements and operations of the forementioned associative ring.

However, our prime motivation for undertaking these studies is not the mere desire to generalize these well-known results of Cartan and Killing from the classical case of matrices to arbitrary simple rings. Rather, the foremost reason for investigating these structures is to provide a tool, a set of techniques, an approach, for answering purely associative questions about the associative structure of these simple rings.

As we proceed in the exposition and development of the researches carried out to date we shall, from time to time, point out how the nonassociative theorems proved can be exploited to prove theorems about simple rings; theorems in whose statements and conclusions these nonassociative structures play no role. These applications, we feel, are but the first ones that have been made. We hope that these techniques will be used advantageously more and more to prove other theorems.

Aside from sketching the use of the results in special cases, we shall give no proofs. These can be found in the appropriate references given.

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