FLOWS ON SOME THREE DIMENSIONAL HOMOGENEOUS SPACES

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1. Flows on surfaces of constant negative curvature have been investigated for some time. The geodesic flow [**3**] and the horocycle flow [**2**] have known minimal and ergodic properties. These flows may be looked at as flows induced on a three dimensional homogeneous space by a one parameter subgroup of a Lie group [**4**]. This idea has been carried further in [**1**; **5**] where one parameter flows on general nilmanifolds are studied.

The manifolds considered here are all compact manifolds of the form G/D where G is a noncompact connected, simply connected three dimensional Lie group and D a discrete uniform subgroup. If $\phi: T \to G$ is a one parameter subgroup of G, then the one parameter flow defined by $t(gD) = \phi(t)gD$, is an action of the reals on G/D. The classification as to which of these flows are minimal and which are ergodic is now complete. In this note we outline this classification; complete proofs will be presented elsewhere.

There are only three cases to consider: simple, nilpotent, and solvable but not nilpotent.

2. G **simple.** If G is simple and noncompact then its Lie algebra $\mathfrak{G}$ is isomorphic to the Lie algebra of the two by two real matrices with trace zero. Each one parameter subgroup of G is of the form $\phi(t) = \exp \overline{X}t$, where $\overline{X} \in \mathfrak{G}$. Let $G(2)$ be the group of all 2×2 real matrices of determinant one. G is the universal covering group of $G(2)$ and we let η be the covering homomorphism $\eta: G \to G(2)$.

THEOREM 1. *If D is a discrete uniform subgroup of G then the mapping $\psi: G/D \to G(2)/\eta(D)$ given by $\psi(gD) = \eta(g)\eta(D)$ is a finite covering and $\eta(D)$ is discrete.*

THEOREM 2. *Let G be the connected, simply connected, noncompact, three dimensional, simple Lie group; and let D be a discrete uniform subgroup of G; and let $\phi(t) = \exp \overline{X}t$. The following statements hold:*

(1) *If $\overline{X}$ has real nonzero eigenvalues the one parameter flow induced*

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