CONICAL SINGULAR POINTS OF DIFFEOMORPHISMS

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1. **Introduction.** The Schoenflies extension Λ_ϕ of a differentiable mapping ϕ, constructed in the proof of Theorem 2.1 of [1], has at most a differential singularity of *conical* type (to be defined). This fact has far-reaching consequences which are reflected in the theorems of [2]. Theorem 1.1 below is one of these consequences. No proof of Theorem 1.1 is given here.

Let S be an $(n-1)$-sphere in a euclidean n-space E and let JS be the closed n-ball in E bounded by S.

THEOREM 1.1. *Let z be an arbitrary point of S. A real analytic diffeomorphism f of S into E admits a homeomorphic extension, F, defined over a set $Z\cup z$, where Z is some open neighborhood of $JS-z$, and $F|Z$ is a real analytic diffeomorphism of Z into E.*

This extension F of f defines an analytic diffeomorphism of its domain of definition with z deleted, and a homeomorphism with z included. F has no singularity on the interior of S, or on S, except at most at z.

We continue with a detailed exposition leading to a proof of Theorem 2.1.

NOTATION. Let E be the euclidean n-space of points (or vectors) x with rectangular coordinates $(x_1, \cdots, x_n)$. Let $\|x\|$ be the distance of x from the origin O. Set

$$S = \{x \mid \|x\| = 1\}. \tag{1.1}$$

If M is a topological $(n-1)$-sphere in E, $\mathring{J}M$ shall denote the open interior of M. The complement of a subset Y of E will be denoted by CY. We use *diff* as an abbreviation of diffeomorphism.

A C_z^m-diff, $m>0$. Let $x\to G(x)$ be a homeomorphism into E of an open neighborhood X of a point $z\in E$; if $G|(X-z)$ is a C^m-diff into E, G will be called a C_z^m-diff of X into E.

An admissible cone K_z. Let K_z be a closed n-cone in E with vertex z, and with sections orthogonal to A which are closed $(n-1)$-balls whose centers are on A. The cone K_z is determined by z, A and any one of its orthogonal sections meeting $A-z$.

A conical point z of G. Let G be a C_z^m-diff into E of an open neighborhood X of z. The point z will be said to be a conical point of G and